CS5232: Formal Specification and Design Techniques

Formal Methods in Action

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Today’s Agenda

A hands-on overview of the tools and techniques

- Using simple examples from other classes
- Not aiming to showcase all features
- Skipping almost all the theory

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The goal:
quick introduction to help you choose a project

The Three Case Studies

Concurrent Readers-Writers Problem in TLA+

- A “Hello World” of concurrent interaction protocols
- Interesting safety and liveness properties
- Focus on a high-level model rather than implementation

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Using SMT solvers for verification and synthesis

- What are SAT and SMT solvers and how are they useful
- Search problems as solutions to constraint systems

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Deductive Verification in Dafny

- Specifying programs in Hoare logics
- Proving that programs do what they should (soundly)

Part I

Specifying Complex Systems in TLA+

Concurrent Reading and Writing

Safe concurrent programs:

Multiple concurrent reads of same memory: *Not* a problem

Multiple concurrent writes of same memory: Problem

Multiple concurrent read & write of same memory: Problem

So far:

If concurrent write/write or read/write might occur,

one can use synchronization to ensure one-thread-at-a-time

But this is unnecessarily conservative:

Could still allow multiple simultaneous readers!

Readers and Writers Problem

variant of the mutual exclusion problem
where there are two classes of processes:

- writers which need exclusive access to resources
- readers which need not exclude each other

Concurrent Correctness

There are two types of correctness properties:

Safety properties

The property must always be true.

Liveness properties

The property must eventually become true.

Exercise: Designing the Protocol for Concurrent Reading and Writing

- What are the components of the system?
- What are its safety properties?
- What about liveness?

Live Demo: Basics of TLA+

- State and variables
- Actions as relations
- Specifying safety and liveness properties
- Detecting and analyzing the violations (bugs in the design!)

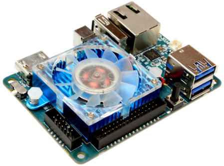
https://github.com/cs5232/basic-examples/

folder “tlaplus”

Part I

SAT and SMT for Verification and Synthesis

The SAT/SMT Revolution



hardware verification

KLEE

Dafny

LiquidHaskell

software verification

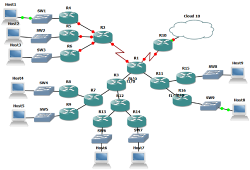
Rosette

Sketch

Leon

Synquid

software synthesis & repair



network configuration synthesis

biological modeling

architecture

Boolean SATisfiability

$$(\text{gin} \lor \text{tonic}) \land (\text{minor} \Rightarrow \neg\text{gin}) \land \text{minor}$$

Boolean SATisfiability

$$(\text{gin} \lor \text{tonic}) \land (\text{minor} \Rightarrow \neg\text{gin}) \land \text{minor}$$

Solution:

$\text{minor} \mapsto T$

$\text{gin} \mapsto F$

$\text{tonic} \mapsto T$

Satisfiability Modulo Theories

$(gin \lor tonic) \land (age < 21 \Rightarrow abv = 0) \land (age = 20)$

Satisfiability Modulo Theories

$$(\text{gin} \lor \text{tonic}) \land (\text{age} < 21 \Rightarrow \text{abv} = 0) \land (\text{age} = 20)$$

In the United States, "gin" is defined as an alcoholic beverage of no less than 40% ABV…

Wikipedia

Satisfiability Modulo Theories

$(gin \lor tonic) \land (age < 21 \Rightarrow abv = 0) \land (age = 20) \land (gin \Rightarrow abv \geq 40)$

In the United States, "gin" is defined as an alcoholic beverage of no less than 40% ABV…

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Satisfiability Modulo Theories

$$(gin \lor tonic) \land (age < 21 \Rightarrow abv = 0) \land (age = 20) \land (gin \Rightarrow abv \geq 40)$$

$age \mapsto 20$

$abv \mapsto 0$

$gin \mapsto F$

$tonic \mapsto T$

Satisfiability Modulo Theories

$$(gin \lor tonic) \land (age < 21 \Rightarrow abv = 0) \land (age = 20) \land (gin \Rightarrow abv \geq 40)$$

theory of Linear Integer Arithmetic

$age \mapsto 20$

$abv \mapsto 0$

$gin \mapsto F$

$tonic \mapsto T$

Popular Solvers

Microsoft	Stanford	SRI	JKU Linz, Austria
Z3	CVC5	Yices2	Boolector

SMT competition: http://smtcomp.sourceforge.net

```
.smt2             // SMTLib format

(declare-fun (Int) age)
(declare-fun (Int) abv)
```

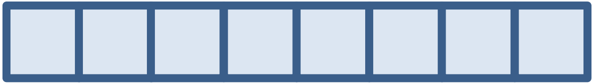
Plan for Today

How to use Z3 for:

1. Constraint programming
2. Program verification
3. Program synthesis

Problem: Array Partitioning

Partition an array of size *N* evenly into *P* sub-ranges



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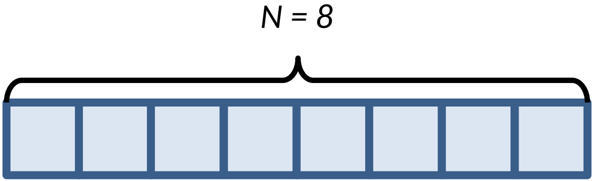

P = 4

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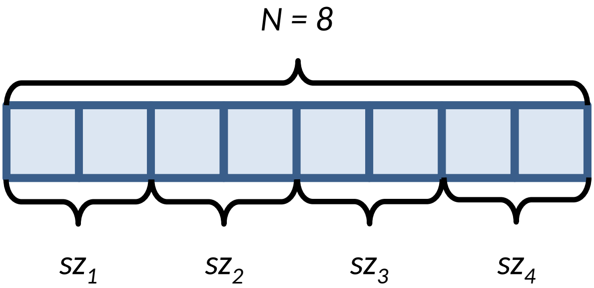

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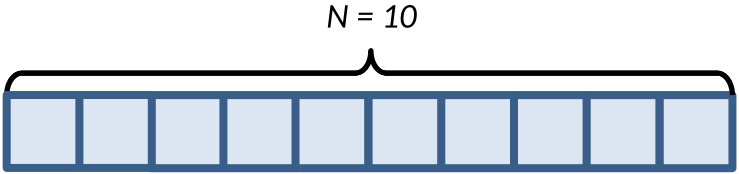

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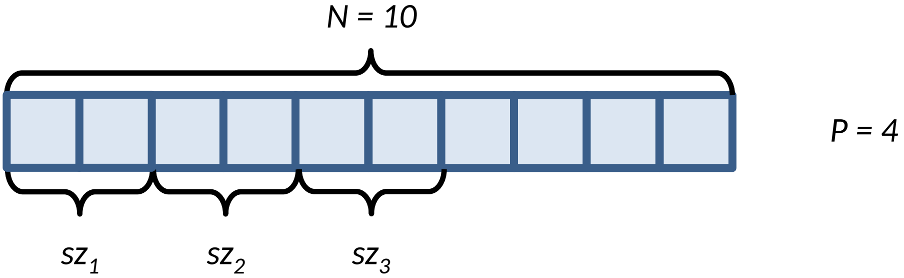

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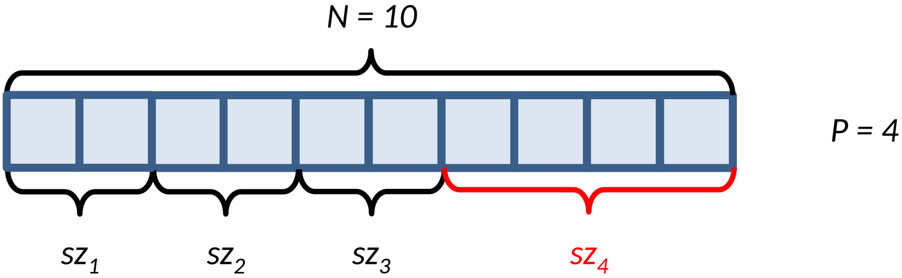

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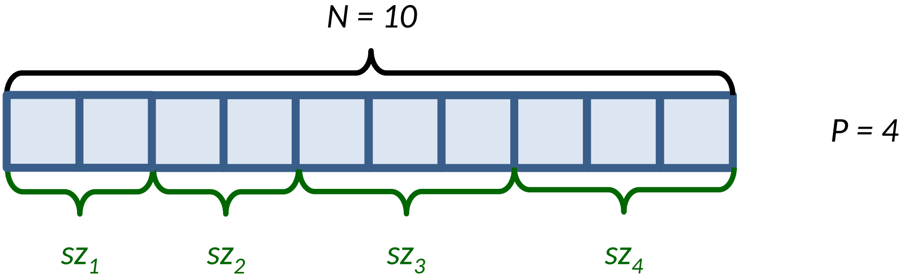

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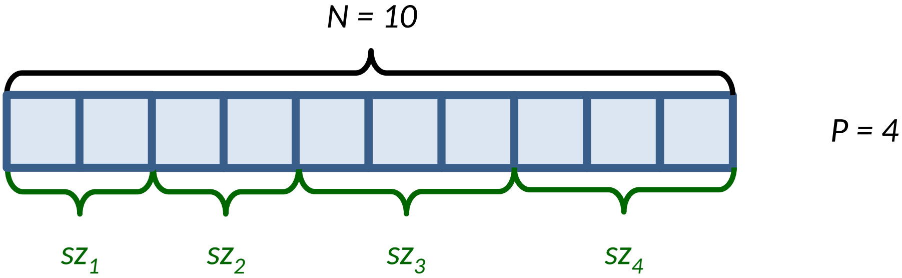

Can we always make them differ by at most 1?

Live Demo

Z3

to the rescue!

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How to use Z3 for:

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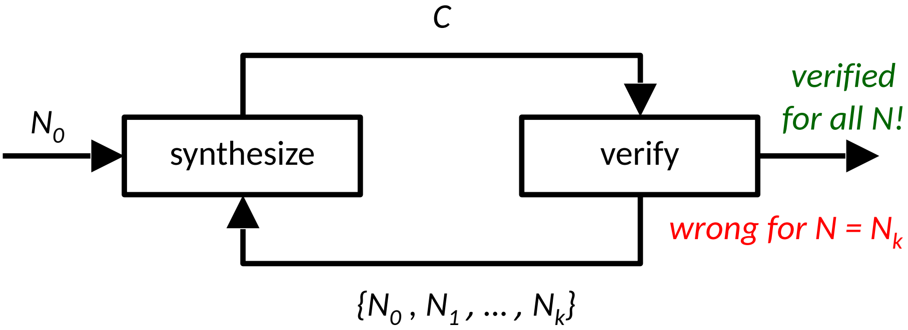
CEGIS



C
N_0
synthesize
verify
verified
for all N!
wrong for N = N_k
$\{N_0, N_1, \ldots, N_k\}$

What we have seen:

Z3

How to use Z3 for:

1. Constraint programming
2. Program verification
3. Program synthesis

https://github.com/cs5232/basic-examples/

folder “smt”

Part III

Deductive Hoare-style Program Verification in Dafny

Program specification

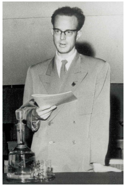
$$\{ P \} \ \mathbf{c} \ \{ Q \}$$

precondition postcondition

Meaning:
If the *initial* state satisfies P,
then the program **c** is safe to run and its ***final*** state satisfies Q.

Example: $\{ \text{True} \}$ $\mathbf{x := 3}$ $\{ x = 3 \}$

Symbolic execution

A method for establishing partial correctness

Independently discovered by ***Robert W. Floyd*** in 1967 and ***Tony Hoare*** in 1969

- also hinted by Turing in 1949;

Also known as ***Hoare-style program logic***, A***xiomatic program semantics***;

Symbolic execution allows us to abstract over specific ***values***

- **e.g.,** instead of x being $1, 2, 3, \ldots$, we can consider input $x \in \mathbb{N} \land x > 0$, reasoning out of these assertions about x;

Specifies ***what*** a program is doing without saying ***how*** it is doing that;

- specifications $\{P\}$ **c** $\{Q\}$ are sometimes called ***Hoare triples***.

Program verification via symbolic execution

Verification is the process of ensuring that the program satisfies the ***specification*** (i.e., pre/postconditions), ascribed to it;

For the purpose of verification, the program is decomposed into ***primitive*** and ***composite*** statements:

Primitive statements are variable ***assignments*** and calls to external functions;

Composite statements are ***conditionals*** (**if-then-else**), **while**-loops and sequential compositions.

Preconditions are assumed/inferred, ***postconditions*** are obtained/checked via ***inference rules*** of symbolic execution.

Inference rules

Assignment

$$\{\, P[e/x] \,\}\ \mathtt{x\ :=\ e}\ \{\, P \,\} \qquad \text{(Assign)}$$

substitute `x` with `e`

Example:

$$\{\, 3 = 3 \,\}\ \mathtt{x\ :=\ 3}\ \{\, x = 3 \,\}$$

Inference rules

Sequential composition

$$\frac{\{\ P\ \}\ c_1\ \{\ Q\ \} \qquad \{\ Q\ \}\ c_2\ \{\ R\ \}}{\{\ P\ \}\ c_1;\ c_2\ \{\ R\ \}}\ \text{(Seq)}$$

Example:

{???} `x := 3; y := x` $\{x = 3 \wedge y = 3\}$

Inference rules

Sequential composition

$$\frac{\{P\}\ c_1\ \{Q\} \qquad \{Q\}\ c_2\ \{R\}}{\{P\}\ c_1; c_2\ \{R\}}\ \text{(Seq)}$$

Example:

$\{3 = 3 \wedge 3 = 3\}$

```
x := 3;
```
(Assign)

$\{x = 3 \wedge x = 3\}$

```
y := x
```
(Assign)

$\{x = 3 \wedge y = 3\}$

Inference rules

Rule of consequence

$$\frac{P \Rightarrow P' \qquad \{ P' \}\ c\ \{ Q' \} \qquad Q' \Rightarrow Q}{\{ P \}\ c\ \{ Q \}} \quad \text{(Conseq)}$$

Example:

$$\{ \text{True} \} \Rightarrow \{3 = 3 \land 3 = 3\}$$

```
x := 3; y := x
```

$$\{x = 3 \land y = 3\}$$

Inference rules

Rule of consequence

$$\frac{P \Rightarrow P' \qquad \{\, P' \,\}\ c\ \{\, Q' \,\} \qquad Q' \Rightarrow Q}{\{\, P \,\}\ c\ \{\, Q \,\}} \ \text{(Conseq)}$$

Example:

$\{\text{True}\}$ `x := 3; y := x` $\{x = 3 \wedge y = 3\}$

Inference rules

Conditional statement

$$\frac{\{\, P \wedge e \,\}\ c_1\ \{\, Q \,\} \qquad \{\, P \wedge \neg e \,\}\ c_2\ \{\, Q \,\}}{\{\, P \,\}\ \textbf{if}\ e\ \textbf{then}\ c_1\ \textbf{else}\ c_2\ \{\, Q \,\}}\ \text{(Cond)}$$

While-loops

$$\frac{\{\, I \wedge e \,\}\ c\ \{\, I \,\}}{\{\, I \,\}\ \textbf{while}\ e\ \textbf{do}\ c\ \{\, I \wedge \neg e \,\}}\ \text{(While)}$$

loop invariant (***needs to be guessed***)

Example for loop invariants

$$\frac{\{ I \wedge e \}\ c\ \{ I \}}{\{ I \}\ \textbf{while}\ e\ \textbf{do}\ c\ \{ I \wedge \neg e \}}$$

Precondition: $\{ x \geq 0 \wedge y = 0 \}$

$\{ x \geq y \wedge y = 0 \} \Rightarrow I$ (Conseq)

```
while (x != y) do {
```

$\{ x \geq y \wedge x \neq y \}$ (While)

$\Rightarrow \{ x > y \}$ (Conseq)

```
    y := y + 1;
```

(Assign)

$\{ x \geq y \} \Rightarrow I$ (Conseq)

```
}
```

$\{ x \geq y \wedge \neg(x \neq y) \}$ (While)

Postcondition: $\Rightarrow \{ x = y \}$ (Conseq)

$e \equiv x \neq y$

Good loop invariant:

$I \equiv x \geq y$

Live Demo

Verifying a program in

Summary

- We have seen three families of tools in action
  - TLA+ for specification and model checking
  - Z3 for constraint solving
  - Dafny for sound logic-based verification
- In the rest of the module, we will learn to use the tools for various applications
- We will also learn about how they work internally