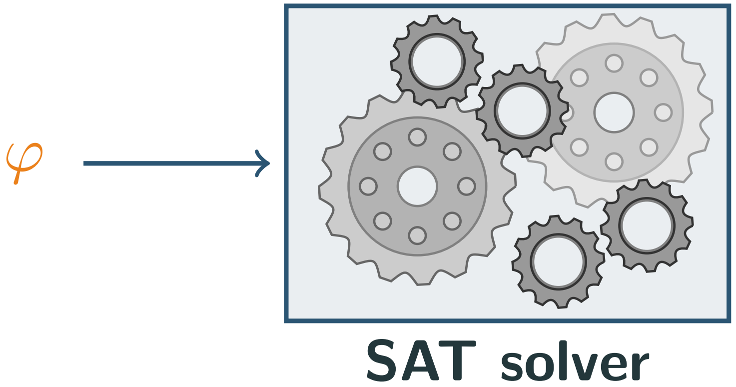
SAT Solving and Its Applications

SAT Solving

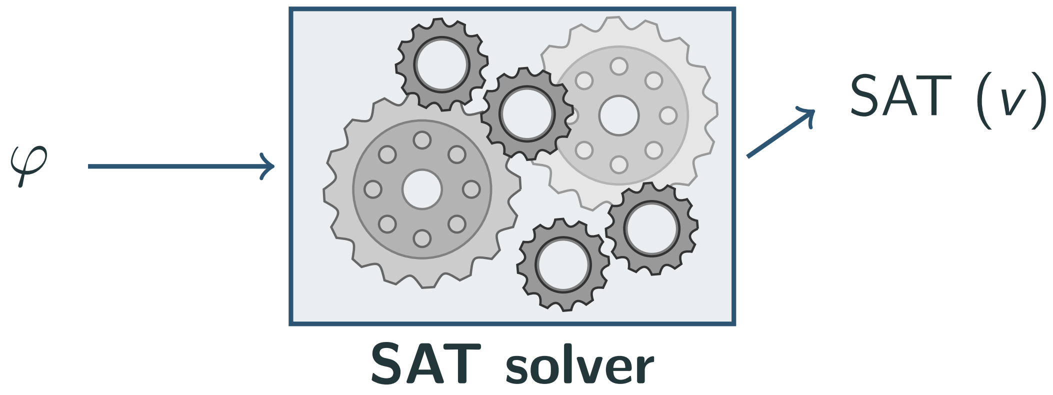
input: propositional formula φ



SAT Solving

input: propositional formula φ

output: SAT + valuation v such that $v(\varphi) = T$ if φ satisfiable



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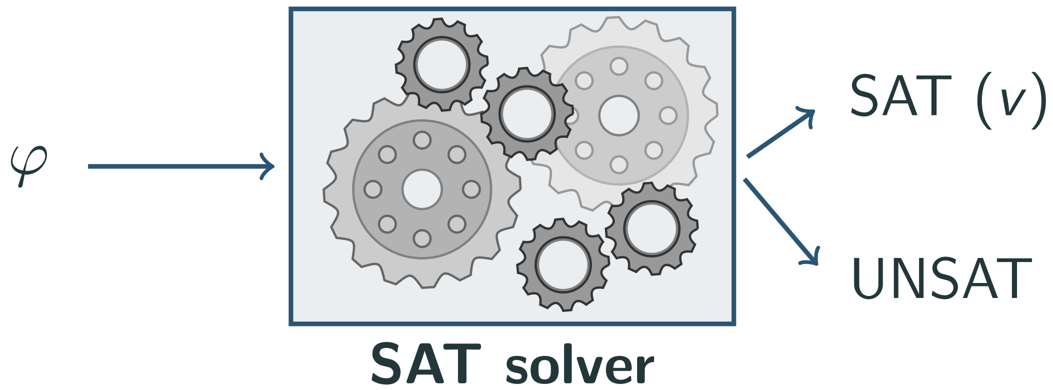
	UNSAT	otherwise



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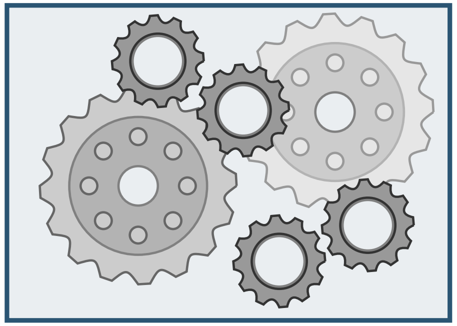
$\varphi \longrightarrow$

$(q \vee \neg r) \wedge (\neg q \vee r) \wedge p$



SAT solver

SAT (v)

$v(p) = \mathrm{T}$
$v(q) = \mathrm{F}$
$v(r) = \mathrm{F}$

UNSAT

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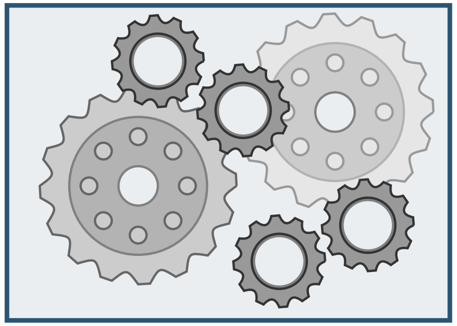

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UNSAT

Terminology

- decision problem P is problem with answer yes or no

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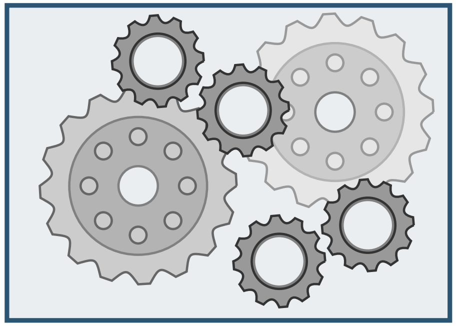

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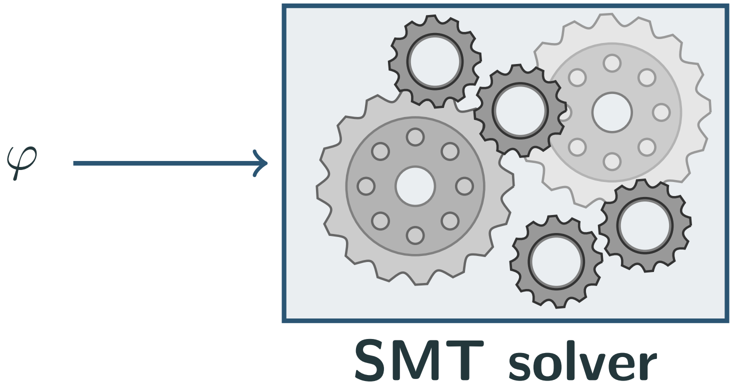
Terminology

- decision problem P is problem with answer yes or no
- SAT encoding of decision problem P is propositional formula φ_P such that

$$\text{answer to } P \text{ is yes} \iff \varphi_P \text{ is satisfiable}$$

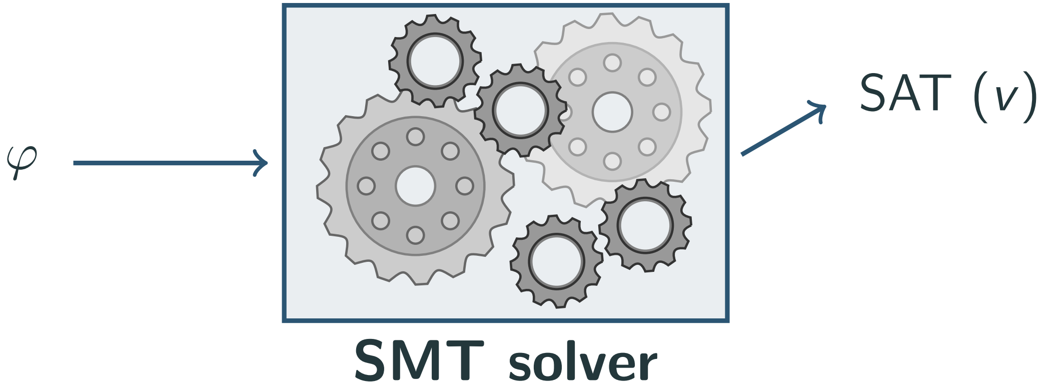
SMT Solving

input: formula φ involving theory T



SMT Solving

input:	formula φ involving theory T	
output:	SAT + valuation v such that $v(\varphi) = T$	if φ is T-satisfiable



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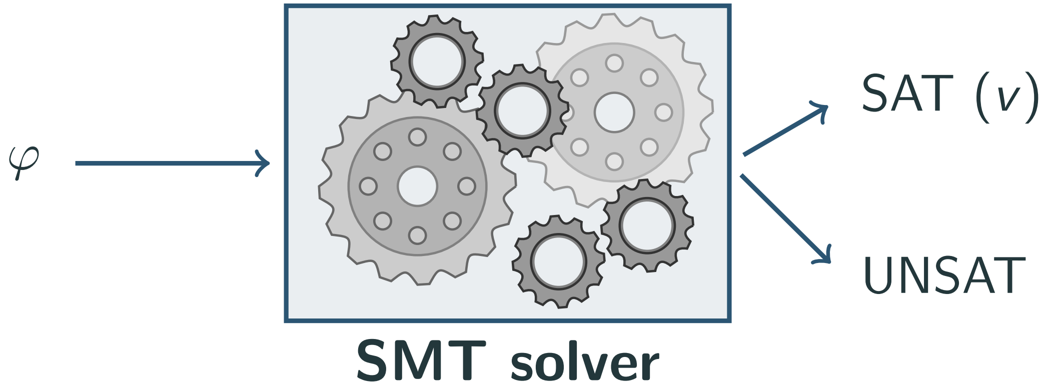

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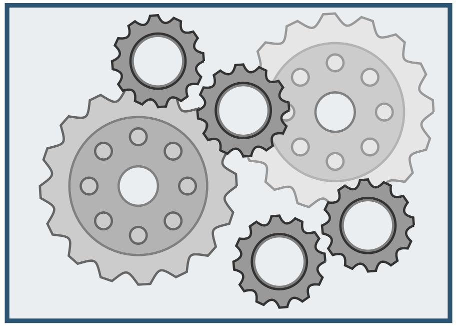
$a + b \geqslant c \vee (a = 0 \wedge p)$



SMT solver

SAT (v) $\quad v(a) = 3 \quad v(b) = 0 \quad v(c) = 0 \quad v(p) = \mathrm{T}$

UNSAT

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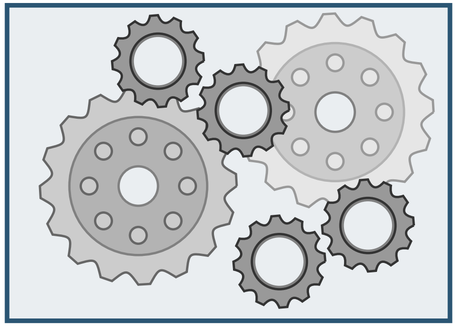

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Example (Theories)

- arithmetic $\qquad 2a + b \geqslant c \vee (a = 0 \wedge p)$

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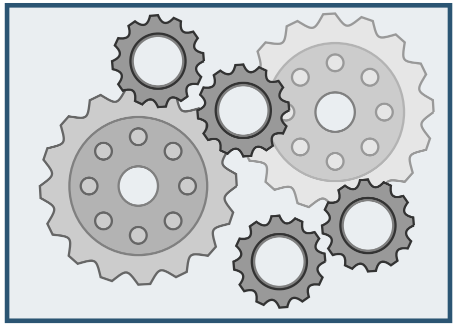

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- arithmetic $2a + b \geqslant c \vee (a = 0 \wedge p)$
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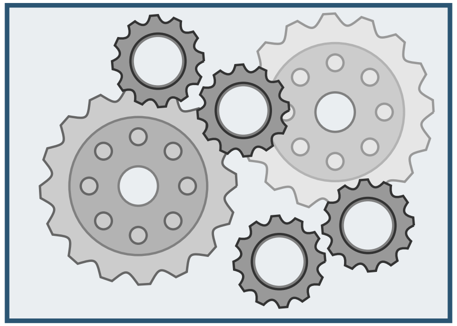

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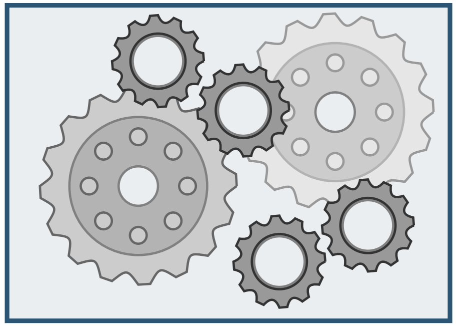

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Terminology

- SMT encoding over theory T of decision problem P is formula φ_P such that

$$\text{answer to } P \text{ is yes} \quad \Longleftrightarrow \quad \varphi_P \text{ is satisfiable}$$

Problem

Austrian driving license test consists of 80 questions out of 1500 such that the following conditions are satisfied:

- 30 questions "main questions" with 3 sub-questions each
- at least 12 main questions must be about crossroads
- at least 12 questions must have pictures
- at least 5 "hard", "medium", and "easy" main questions

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Result

easy generation of valid question sets (with some random preselection)

Application: Pythagorean Triples

Problem

Can one color all natural numbers with two colors such that whenever $x^2 + y^2 = z^2$ not all of x, y, and z have same color?

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$$3^2 + 4^2 = 5^2 \qquad 5^2 + 12^2 = 13^2$$

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Result: No. Coloring exists only up to 7,825.

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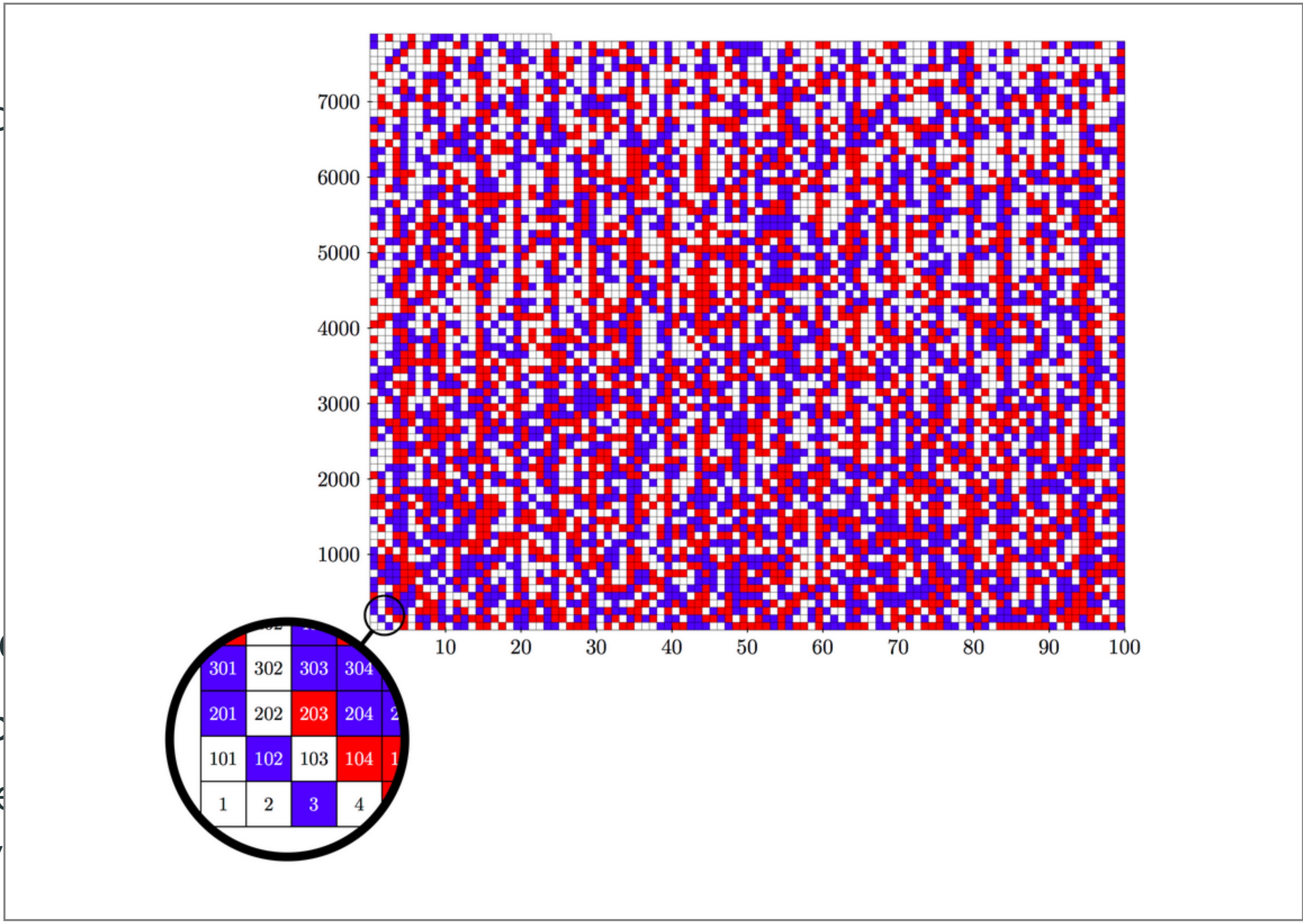

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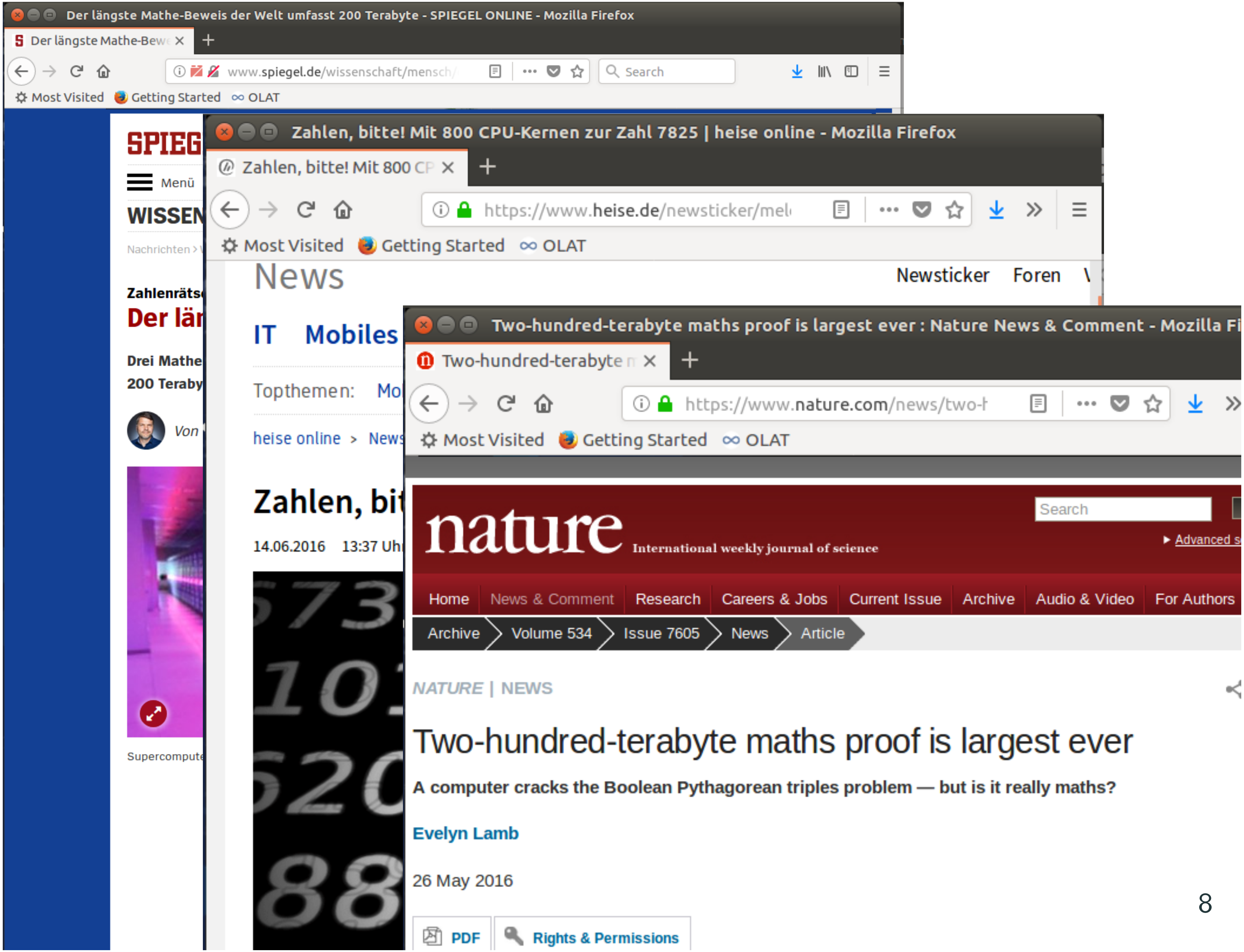
1000s of variables, solving time 2 days with 800 processors, 200 TB of proof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Evelyn Lamb
26 May 2016
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Application: Tournament Scheduling

Problem: Round Robin Scheduling

Schedule sports league tournament for n teams, p periods of $n-1$ rounds each
(+ venue restrictions, break restrictions, $\ldots$)

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Example (Österreichische Fußball-Bundesliga)

10 teams play in 4 periods (9 rounds each), periods 1 & 2 and 3 & 4 mirrored

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Result

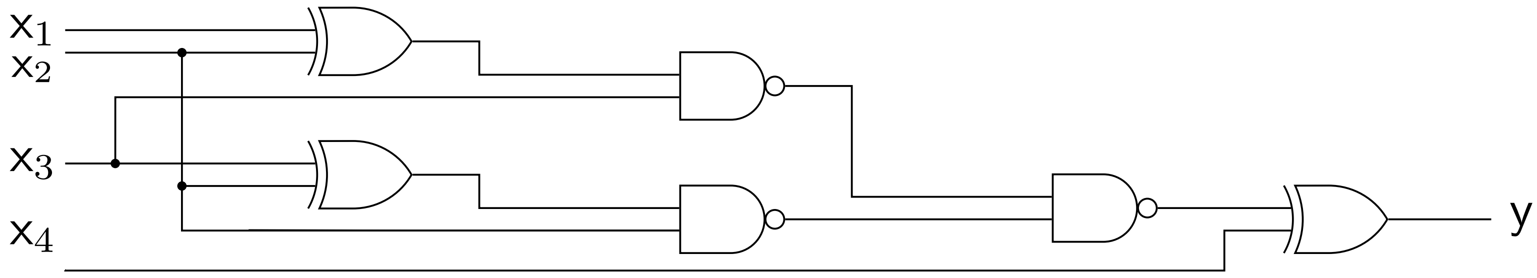
SAT scheduling is 100x faster than previous industrial scheduling tools

Application: Hardware Verification

Problem

- errors in hardware chips are costly (Intel paid $475 million for FDIV bug)

Example (Formal Circuit Model)



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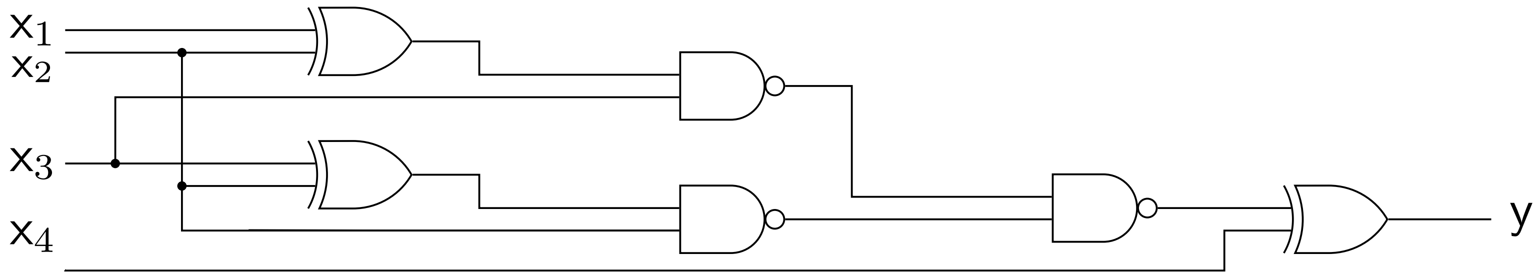

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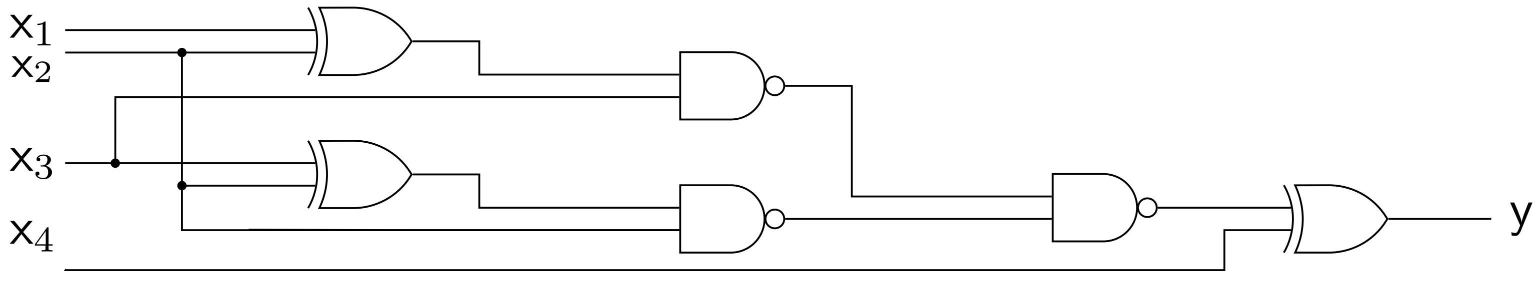

SAT Encoding

- variables for input and output
- SAT formulas for implemented behavior and expected behavior (specification)
- check for equivalence

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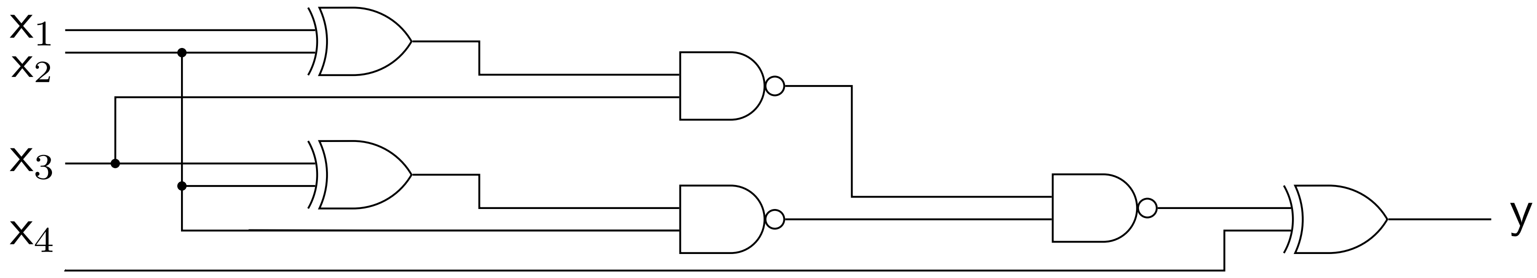

SAT Encoding

- variables for input and output
- SAT formulas for implemented behavior and expected behavior (specification)
- check for equivalence

Impact

- ensured correctness, more reliable hardware components (formal verification)
- manufacturers rely on SAT-based verification since beginning of 2000s
  e.g., Intel Core i7 implements over 2700 distinct verified microinstructions

Propositional Logic Revisited

Concepts

- literal
- formula
- assignment
- satisfiability and validity
- negation normal form (NNF)
- conjunctive normal form (CNF)
- disjunctive normal form (DNF)

Definition (Propositional Logic: Syntax)

propositional formulas are built form

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- $\rightarrow$, $\wedge$, $\vee$ are right-associative: $p \rightarrow q \rightarrow r$ denotes $p \rightarrow (q \rightarrow r)$

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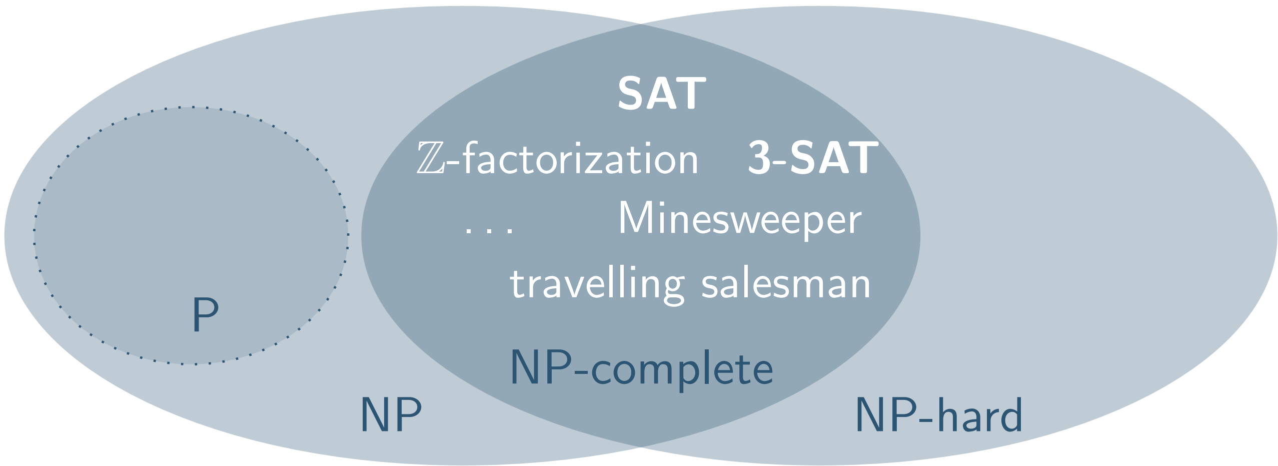

- 1 million \$ prize money awarded for solution to $\mathbf{P} \stackrel{?}{=} \mathbf{NP}$

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Example

$$p \vee q \;\approx\; \top \qquad\qquad p \wedge \neg p \;\approx\; q \wedge \neg q \qquad\qquad p \wedge \neg p \;\not\approx\; p \wedge \neg q$$

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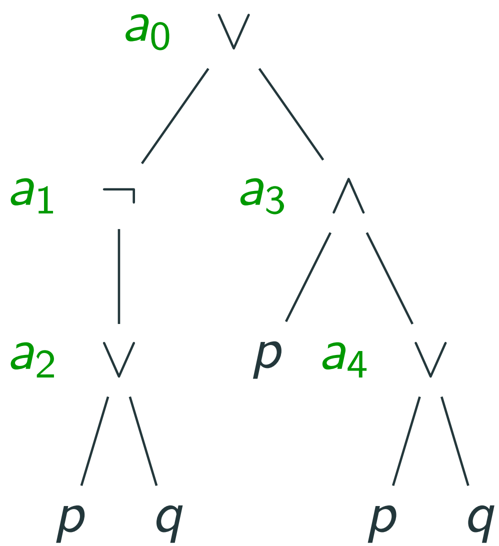

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- $\varphi = \neg(p \vee q) \vee (p \wedge (p \vee q))$
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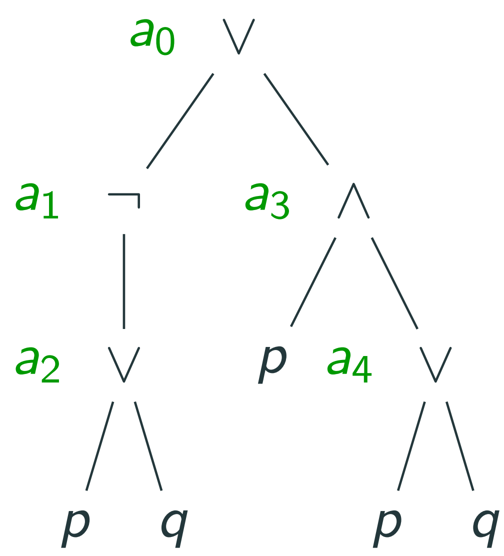

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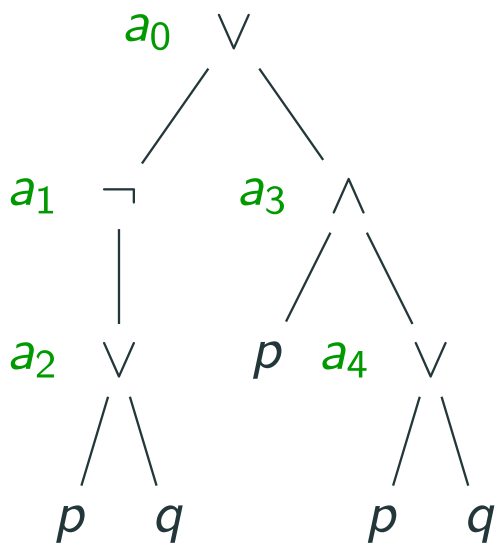

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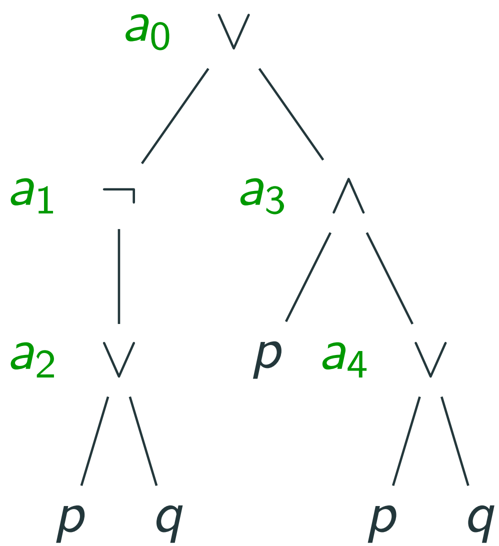

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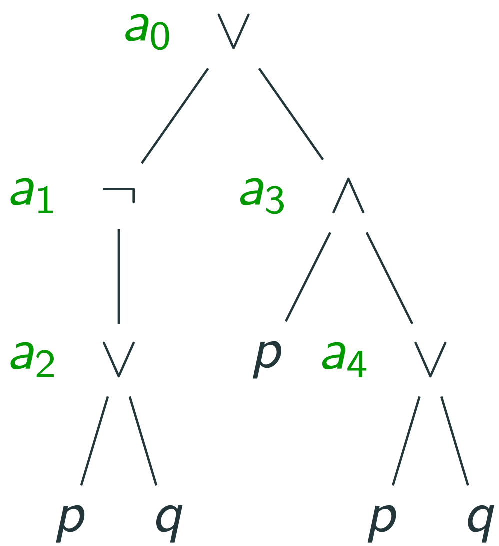

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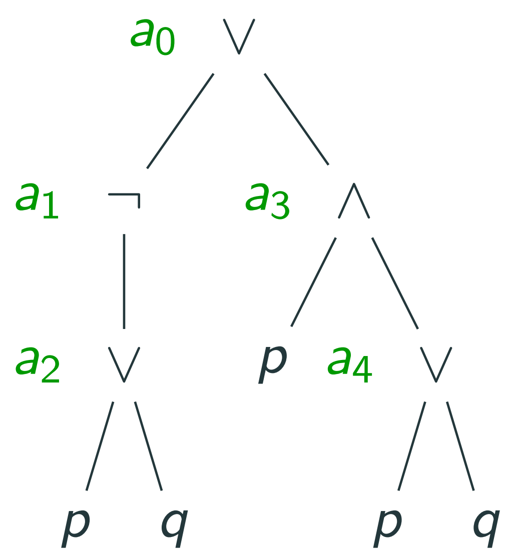
- common subexpressions can be shared

Example (Tseitin's Transformation)



- $\varphi = \neg(p \vee q) \vee (p \wedge (p \vee q))$
- use fresh propositional variable for every connective

  $a_0 : \neg(p \vee q) \vee (p \wedge (p \vee q))$ $\quad a_1 : \neg(p \vee q)$

  $a_2 : p \vee q$ $\quad a_3 : p \wedge (p \vee q)$

  $a_4 : p \vee q$

- $\varphi \approx \quad a_0 \wedge (a_0 \leftrightarrow a_1 \vee a_3) \wedge (a_1 \leftrightarrow \neg a_2) \wedge (a_2 \leftrightarrow p \vee q) \wedge (a_3 \leftrightarrow p \wedge a_2)$
- every $\leftrightarrow$ subexpression can be replaced by at most three clauses:

$$
\begin{aligned}
a \leftrightarrow b \wedge c &\quad\equiv\quad (\neg a \vee b) \wedge (\neg a \vee c) \wedge (a \vee \neg b \vee \neg c) \\
a \leftrightarrow b \vee c &\quad\equiv\quad (\neg a \vee b \vee c) \wedge (a \vee \neg b) \wedge (a \vee \neg c) \\
a \leftrightarrow \neg b &\quad\equiv\quad (\neg a \vee \neg b) \wedge (a \vee b)
\end{aligned}
$$

- common subexpressions can be shared

SAT Solvers

Minisat, Glucose, CaDiCaL, Glu_VC, Plingeling, MapleLRB LCM, MapleCOMPSPS, Riss, Lingeling, Treengeling, CryptoMiniSat, abcdSAT, Dimetheus, Kiel, MapleCOMSPS, Rsat, SWDiA5BY, BlackBox, SWDiA5BY, pprobSAT, glueSplit_clasp, BalancedZ, SApperloT, PeneLoPe, MXC, ROKKminisat, MiniSat_HACK_999ED, ZENN, CSHCrandMC, MiniGolf, march_rw, sattime2011, mphasesat64, sparrow2011, pmcSAT, CSHCpar8, gluebit_clasp, clasp, precosat, gNovelty, SATzilla, SatELite, Score2SAT, YalSAT, tch glucose3, . . .

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- annual competition for different tracks (main, parallel, no-limit, ...)
- increasing set of benchmarks from industry, mathematics, cryptography, ...
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Minisat

- minimalistic open source solver (`http://minisat.se/` or apt, yum,. . .)

```
$ minisat test.sat result.txt
```

- web interface

Example (DIMACS)

formula $(x_1 \vee \neg x_3) \wedge (x_2 \vee x_3 \vee \neg x_1) \wedge (\neg x_1 \vee x_2 \vee x_4)$ can be expressed by

```
c a very simple example
p cnf 4 3
1 -3 0
2 3 -1 0
-1 2 4 0
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- lines starting with `c` are considered comments