CS5232: Formal Specification and Design Techniques

Reasoning about Arrays and Objects in Dafny

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Arrays are references

```
var a := new string[20];
a[7] := "hello";
var b := a;
assert b[7] == "hello";

b[7] := "hi";
a[8] := "greetings";
assert a[7] == "hi" && b[8] == "greetings";
```

Type of a is `array<string>`

Arrays are references

```
var a := new string[20];
a[7] := "hello";
var b := a;
assert b[7] == "hello";

b[7] := "hi";
a[8] := "greetings";
assert a[7] == "hi" && b[8] == "greetings";

b := new string[8];
b[7] := "long time, no see";
assert a[7] == "hi";
assert a.Length == 20 && b.Length == 8;
```

Type of a is
`array<string>`

Multi-dimensional arrays

```
var m := new bool[3, 10];
m[0, 9] := true;
m[1, 8] := false;
assert m.Length0 == 3 && m.Length1 == 10;
```

Type of m is
`array2<bool>`

Sequences

Arrays are mutable and are *reference types*

Sequences are immutable and are *value types*, like bool and int

To declare a sequence we use type constructor seq,
e.g., seq<bool>, seq<int>

Examples:

[]	the empty sequence
[58]	singleton integer sequence
["hey", "hola", ”salut"]	string sequence

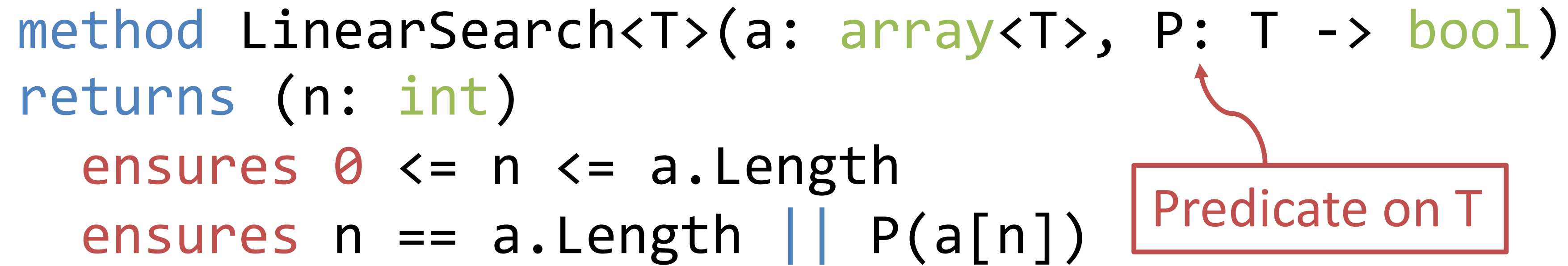
Sequences

```
var s := [6, 28, 496];
assert s[2] == 496;
assert |s| == 3;   // length function
assert s + [8128] == [6, 28, 496, 8128];

var p := [1, 5, 12, 22, 35]
assert p[2..4] == [12, 22];
assert p[..2] == [1, 5];
assert p[2..] == [12, 22, 35];

a := new int[3];
a[0], a[1], a[2] := 6, 28, 496;
s, p := a[..], a[..2];
assert s == [6, 28, 496] && p == [6, 28];
```

Linear search

```
method LinearSearch<T>(a: array<T>, P: T -> bool)
returns (n: int)
```

Linear search



Linear search

```
method LinearSearch<T>(a: array<T>, P: T -> bool)
returns (n: int)
  ensures 0 <= n <= a.Length
  ensures n == a.Length || P(a[n])
{
  n := 0;
  while n != a.Length
    invariant 0 <= n <= a.Length



}
```

Linear search

```
method LinearSearch<T>(a: array<T>, P: T -> bool)
returns (n: int)
  ensures 0 <= n <= a.Length
  ensures n == a.Length || P(a[n])
{
  n := 0;
  while n != a.Length
    invariant 0 <= n <= a.Length
  {
    if P(a[n])
      { return; }
    n := n + 1;
  }
}
```

return jumps to end of method, and we need to prove the postcondition

Alternative implementation

```
method LinearSearch1<T>(a: array<T>, P:T -> bool)
returns (n: int)
  ensures 0 <= n <= a.Length
  ensures n == a.Length || P(a[n])
{
  n := a.Length;
}
```

Alternative implementation

```
method LinearSearch1<T>(a: array<T>, P:T -> bool)
returns (n: int)
  ensures 0 <= n <= a.Length
  ensures n == a.Length || P(a[n])
{
  n := a.Length;
}
```

To specify that no elements satisfy P, when `n == a.Length` we need to quantify over the elements of a.

We can achieve the same effect by quantifying over the array positions instead:

```
forall i :: 0 <= i < a.Length ==> !P(a[i])
```

Strengthening the contract

```
method LinearSearch1<T>(a: array<T>, P:T -> bool)
returns (n: int)
  ensures 0 <= n <= a.Length
  ensures n == a.Length || P(a[n])
  ensures n == a.Length ==>
            forall i :: 0 <= i < a.Length ==> !P(a[i])
```

Strengthening the contract

```
method LinearSearch1<T>(a: array<T>, P:T -> bool)
returns (n: int)
  ensures 0 <= n <= a.Length
  ensures n == a.Length || P(a[n])
  ensures n == a.Length ==>
            forall i :: 0 <= i < a.Length ==> !P(a[i])
```

We use the “replace a constant by a variable” loop design technique seen before:

Linear search

```
{ forall i :: 0 <= i < n + 1 ==> ! P(a[i]) }
n := n + 1;
{ forall i :: 0 <= i < n ==> ! P(a[i]) }
```

Linear search

```
{ forall i :: 0 <= i < n || i == n ==> !P(a[i]) }
{ forall i :: 0 <= i < n + 1 ==> ! P(a[i]) }
n := n + 1;
{ forall i :: 0 <= i < n ==> ! P(a[i]) }
```

Linear search

```
forall x :: A || B ==> C
= (forall x :: A ==> C) && (forall x :: B ==> C)
```

```
{ (forall i :: 0 <= i < n ==> ! P(a[i])) &&
  (forall i :: i == n ==> ! P(a[i]))
}
{ forall i :: 0 <= i < n || i == n ==> !P(a[i]) }
{ forall i :: 0 <= i < n + 1 ==> ! P(a[i]) }
n := n + 1;
{ forall i :: 0 <= i < n ==> ! P(a[i]) }
```

Linear search

```
(forall x :: x == E ==> A) = A[x\E]
```
(one-point rule)

```
{ (forall i :: 0 <= i < n ==> !P(a[i])) && !P(a[n]) }
{ (forall i :: 0 <= i < n ==> ! P(a[i])) &&
  (forall i :: i == n ==> ! P(a[i]))
}
{ forall i :: 0 <= i < n || i == n ==> !P(a[i]) }
{ forall i :: 0 <= i < n + 1 ==> ! P(a[i]) }
n := n + 1;
{ forall i :: 0 <= i < n ==> ! P(a[i]) }
```

Linear search

holds due to invariant

```
{ (forall i :: 0 <= i < n ==> !P(a[i])) && !P(a[n]) }
{ (forall i :: 0 <= i < n ==> ! P(a[i])) &&
  (forall i :: i == n ==> ! P(a[i]))
}
{ forall i :: 0 <= i < n || i == n ==> !P(a[i]) }
{ forall i :: 0 <= i < n + 1 ==> ! P(a[i]) }
n := n + 1;
{ forall i :: 0 <= i < n ==> ! P(a[i]) }
```

holds after `if (P(a[n])) { return; }`
(i.e., now we know that !P(a[n])) holds)

Linear search

```
{ (forall i :: 0 <= i < n ==> !P(a[i])) && !P(a[n]) }
{ (forall i :: 0 <= i < n ==> ! P(a[i])) &&
  (forall i :: i == n ==> ! P(a[i]))
}
{ forall i :: 0 <= i < n || i == n ==> !P(a[i]) }
{ forall i :: 0 <= i < n + 1 ==> ! P(a[i]) }
n := n + 1;
{ forall i :: 0 <= i < n ==> ! P(a[i]) }
```

Loop body for `LinearSearch` works here

Full program

```
method LinearSearch1<T>(a: array<T>, P:T -> bool)
returns (n: int)
  ensures 0 <= n <= a.Length
  ensures n == a.Length || P(a[n])
  ensures n == a.Length ==>
               forall i :: 0 <= i < a.Length ==> !P(a[i])
{
  n := 0;
  while n != a.Length
    invariant 0 <= n <= a.Length
    invariant forall i :: 0 <= i < n ==> !P(a[i])
  {
    if P(a[n]) { return; }
    n := n + 1;
  }
}
```

Reading arrays in functions

If a function/predicate accesses the elements of an input array a*,*
its specification must include reads a

```
function IsZeroArray(a: array<int>, lo: int, hi: int): bool
  requires 0 <= lo <= hi <= a.Length
  reads a
  decreases hi - lo
{
  lo == hi || (a[lo] == 0 && IsZeroArray(a, lo + 1, hi))
}
```

Modifying arrays

If a method modifies values accessible through reference parameters (and stored in the heap),
its specification must identify the relevant parts of the heap using frames

```
method SetEndPoints(a: array<int>, left: int, right: int)
  requires a.Length != 0
  modifies a
{
  a[0] := left;
  a[a.Length - 1] := right;
}
```

modifies clause

If a method changes the elements of an input array a*,*
its specification must include modifies a

```
method Aliases(a: array<int>, b: array<int>)
  requires 100 <= a.Length
  modifies a
{
  a[0] := 10;
  var c := a;
  if b == a {
    b[10] := b[0] + 1;   // ok since b == a
  }
  c[20] := a[14] + 2;    // ok since c == a
}
```

old qualifier

The expression `old(E)` *denotes the value of* `E` *on entry to the enclosing method*

```
method UpdateElements(a: array<int>)
  requires a.Length == 10
  modifies a
  ensures old(a[4]) < a[4]
  ensures a[6] <= old(a[6])
  ensures a[8] == old(a[8])
{
  a[4], a[8] := a[4] + 3, a[8] + 1;
  a[7], a[8] := 516, a[8] - 1;
}
```

old qualifier

`old` *affects only the heap dereferences in its argument*

For example, in

```
method OldVsParameters(a: array<int>, i: int)
returns (y: int)
  requires 0 <= i < a.Length
  modifies a
  ensures old(a[i] + y) == 25
```

only `a` is interpreted in the pre-state of the method (but not **`y`**)

New arrays

A method is allowed to allocate a new array and change its elements without mentioning the array in the `modifies` *clause*

```
method NewArray() returns (a: array<int>)
  ensures a.Length == 20
{
  a := new int[20];
  var b := new int[30];
  a[6] := 216;
  b[7] := 343;
}
```

Fresh arrays

```
method Caller()
{
  var a := NewArray();
  a[8] := 512;     // error: modification of a not allowed
}
```

To fix error, strengthen specification of `NewArray` to

```
method NewArray() returns (a: array<int>)
  ensures fresh(a)
  ensures a.Length == 20
```

Initializing arrays

```
method InitArray<T>(a: array<T>, d: T)
  modifies a
  ensures forall i :: 0 <= i < a.Length ==> a[i] == d
{
  var n := 0;
  while n != a.Length
    invariant 0 <= n <= a.Length
    invariant forall i :: 0 <= i < n ==> a[i] == d
  {
    a[n] := d;
    n := n + 1;
  }
}
```

Incrementing the values in an array

```
method IncrementArray(a: array<int>)
  modifies a
  ensures forall i :: 0 <= i < a.Length ==> a[i] == old(a[i]) + 1
{
  var n := 0;
  while n != a.Length
    invariant 0 <= n <= a.Length
    invariant forall i :: 0 <= i < n ==> a[i] == old(a[i]) + 1

  {
    a[n] := a[n] + 1;
    n := n + 1;
  }  // error: second loop invariant not maintained (why?)
}
```

Incrementing the values in an array

```
method IncrementArray(a: array<int>)
  modifies a
  ensures forall i :: 0 <= i < a.Length ==> a[i] == old(a[i]) + 1
{
  var n := 0;
  while n != a.Length
    invariant 0 <= n <= a.Length
    invariant forall i :: 0 <= i < n ==> a[i] == old(a[i]) + 1
    invariant forall i :: n <= i < a.Length ==> a[i] == old(a[i]) // needed
  {
    a[n] := a[n] + 1;
    n := n + 1;
  }
}
```

We need to add the invariant that elements not yet visited maintain the old value

Copying arrays

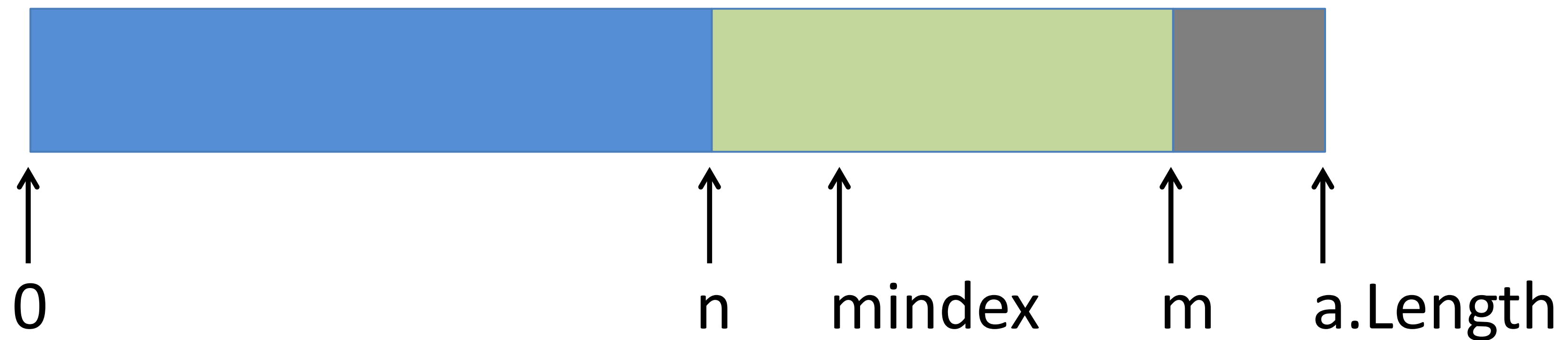
```
method CopyArray<T>(a: array<T>, b: array<T>)
  requires a.Length == b.Length
  modifies b
  ensures forall i :: 0 <= i < a.Length ==> b[i] == old(a[i])
{
  var n := 0;
  while n != a.Length
    invariant 0 <= n <= a.Length
    invariant forall i :: 0 <= i < n ==> b[i] == old(a[i])
    invariant forall i ::
                0 <= i < a.Length ==> a[i] == old(a[i])
  { b[n] := a[n];
    n := n + 1;
  }
}
```

Minimum

- See the code *Min.dfy*

Selection sort

- See the code *SelectionSorty.dfy*



QuickSort

- See the code *QuickSort.dfy*

Reasoning about Objects

Checksum Objects

An *object* is an instance of a *class*, and like arrays, has a *reference type*

```
class ChecksumMachine {
  var data: string

  constructor ()
    ensures data == ""

  method Append(d: string)
    modifies this
    ensures data == old(data) + d

  function Checksum(): int
    reads this
    ensures Checksum() == Hash(data)
…
```

string is shorthand for seq<char>

Checksum Objects

```
  …

  function Hash(s: string): int {
    SumChars(s) % 137
  }

  function SumChars(s: string): int {
    if |s| == 0 then 0 else
      var last := |s| - 1;
      SumChars(s[..last]) + s[last] as int
  }
}
```

Test client

```
method Main() {
   var m := new ChecksumMachine();
   m.Append("green ");
   m.Append("grass");
   var c := m.Checksum();
   print "Checksum is ", c, "\n";
}
```

A method is allowed to allocate new arrays and objects and change their state (that is, the elements of the arrays and the fields of the objects) without mentioning these arrays and objects in the `modifies` *clause*

Class Invariant

To write efficient implementation, want to keep track of checksum so far:

```
var cs: int
```

We want to use `data` in specifications, but not in compiled program:

```
ghost var data: string

predicate Valid(d)
  reads this
{ cs == Hash(d) }
```

A predicate is a Boolean function

If a function accesses the fields of an object `o`*, its specification must include* `reads o`

Class Invariant

```
class ChecksumMachine {
  ghost var data: string

  predicate Valid()
    reads this

  constructor ()
    ensures Valid(data) && data == ""

  method Append(d: string)
    requires Valid(data)
    modifies this
    ensures Valid data && data == old(data) + d

  function method Checksum(): int
    requires Valid(data)
    reads this
    ensures Checksum() == Hash(data)
}
```

Implementation

```
constructor ()
   ensures Valid(data) && data == ""
{ cs := 0;
  data := ""
}
```

A constructor is allowed to assign to the fields of the object being constructed, `this`*, without mentioning* `this` *in the* `modifies` *clause*

```
function method Checksum(): int
  requires Valid(data)
  reads this
  ensures CheckSum() == Hash(data)
{  cs  }
```

Implementation

```
method Append(d: string)
  requires Valid(data)
  modifies this
  ensures Valid(data)
  ensures data == old(data) + d
{
  var i := 0;
  while i != |d|
    invariant 0 <= i <= |d|
    invariant Valid(data)
    invariant data == old(data) + d[..i]
  {
    cs := (cs + d[i] as int) % 137;
    data := data + [d[i]];
    i := i + 1;
  }
}
```

Summary

- Verification Dafny works by means of computing *weakest preconditions* and discharging proof obligations via SMT
- Loops require explicit *invariants* and *termination measures* (aka *variants*, *ranking functions*)
- Object invariants should be preserved by each method