

# Practical Formal Methods

## Course Overview and Introduction

Ilya Sergey

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[ilyasergey.net/PFM24](http://ilyasergey.net/PFM24)

# About myself

- Undergrad** [Saint Petersburg State University](#), 2008
- PhD** [KU Leuven](#), 2012
- Currently** Associate Professor at [NUS School of Computing](#) (since 2018)
- Previously** Assistant Professor at [University College London](#)  
Postdoc at [IMDEA Software Institute](#)  
Software Engineer at [JetBrains](#) (IntelliJ IDEA team: Scala, Groovy)
- Research interests** software verification, PL design, concurrent & distributed algorithms

[ilyasergey.net](http://ilyasergey.net)

# Course Info and Material

- All information, including the syllabus, available on **website** at:  
<https://ilyasergey.net/PFM24/>
- Textbooks:
  - *Specifying Systems* by Leslie Lamport, 2002
  - *Program Proofs* by Rustan Leino, 2020
- Class notes and additional reading material to be posted on the website
- Announcements, submissions and grades on Telegram
- Accompanying code on GitHub (send me your GH handle to get access!):

<https://github.com/formal-and-practical>

# Goals of the Course

1. Learn about formal methods (FMs) in system design and software engineering
2. Understand how FMs help produce high-quality software
3. Learn about formal modelling and specification languages
4. Write and understand formal requirement specifications
5. Learn about main approaches in formal software verification
6. Learn about underpinning for state-of-the-art verification tools
7. Use automated and interactive tools to verify models and code

# Course Topics

## Software Specification and Validation

- High-level system design
- Foundations of automated reasoning
- Code-level design

## Main Software Validation Techniques

**Model Checking:** often automatic, unsound

**Decidable Reasoning:** reducing verification to known algorithmic problems

**Deductive Verification:** typically semi-automatic, precise (source code level)

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**Deductive Verification:** typically semi-automatic, precise (source code level)

Abstract Interpretation: automatic, correct, incomplete, terminating

Practical tools  
we will learn

# Part I: High-Level Design



## Language: TLA+

- Lightweight modelling language for system design
- Amenable to a fully automatic analysis
- Aimed at expressing complex behaviour and properties of a software system
- Intuitive structural modelling tool based on Boolean functions
- Automatic analyser based on bounded model checking

## Learning Outcomes

- Design and model software systems in the TLA+ language
- Check models and their properties with the TLC model checker
- Understand the practical limitations of TLA+



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## Part II: Logical Foundations



### Language: SAT and SMT formulas

- Basic formalism for encoding systems and their properties
- Foundation of most of existing verification techniques
- Typically, not used explicitly but rather as a compilation target
- Puts strict constraints on expressivity

### Learning Outcomes

- Identify problems that can be encoded as SAT or SMT
- Encode decidable verification and synthesis problems
- Using state of the art solvers, such as Z3 and CVC4

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- Encode decidable verification and synthesis problems
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## Part III: Code-level Specification



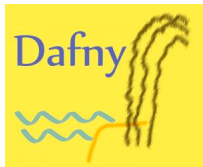
### Language: Dafny

- Programming language with specification constructs
- Specifications embedded in source code as formal contracts
- Tool support with sophisticated verification engines
- Automated analysis based on theorem proving techniques

### Learning Outcomes:

- Write formal specifications and contracts in Dafny
- Verify functional properties of Dafny programs with automated tools
- Understand what can and cannot be expressed in Dafny

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# Assessment

## **Homework Assignments: 30%**

- Homework 1: TLA+: 20%
- Homework 2: SMT: 20%
- Homework 3: Dafny: 20%

## **Research Project: 40%**

- Done in teams of one or two
- Includes implementation, written report, and presentation
- Part of the score is by means of self- and peer assessment

# Introduction

# Today's reality

## Software has become critical to modern life

- **Communication** (internet, voice, video, ...)
- **Transportation** (air traffic control, avionics, cars, ...)
- **Health Care** (patient monitoring, device control, ...)
- **Finance** (automatic trading, banking, ...)
- **Defense** (intelligence, weapons control, ...)
- **Manufacturing** (precision milling, assembly, ...)
- **Process Control** (oil, gas, water, ...)
- ...



# Embedded Software

Software is now embedded everywhere

Some of it is critical

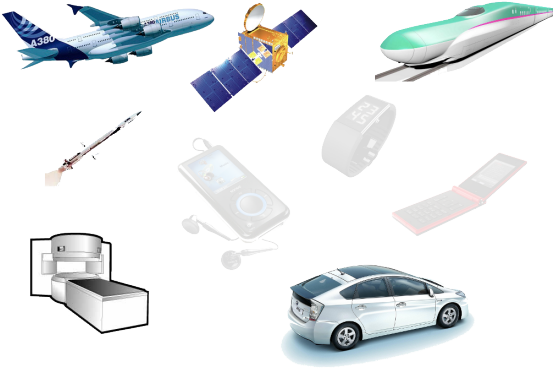


Failing software costs money and life!

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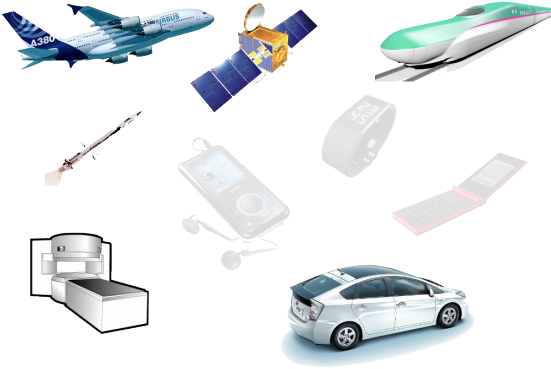


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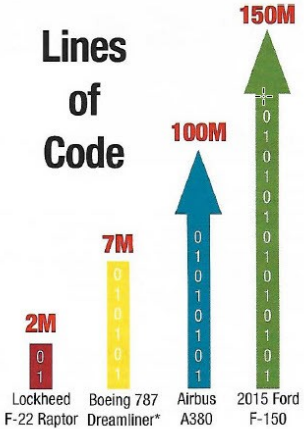
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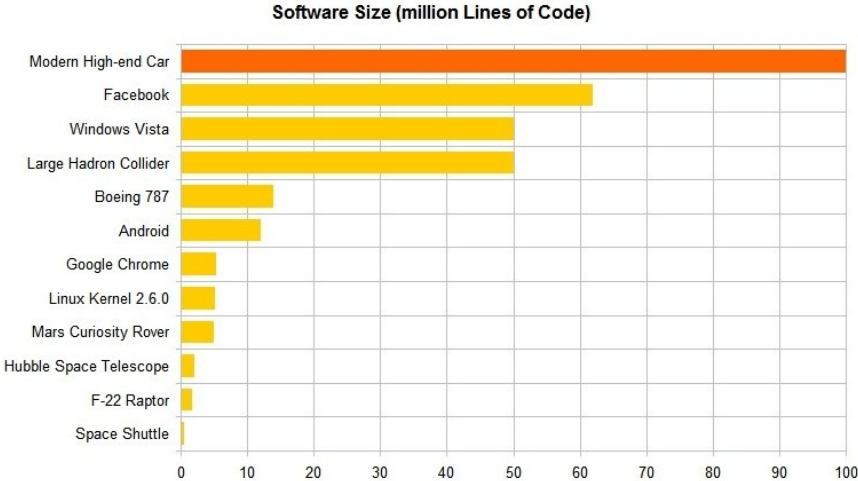
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# Software Systems are Growing Very Large



\* Avionics and online support systems only.

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A typical 2022 car model contains >100M lines of code  
How do you verify that?

Current cars admit hundreds of onboard functions  
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# Failing Software Costs Money

Expensive recalls of products with embedded software

Lawsuits for loss of life or property damage

- Car crashes (e.g., Toyota Camry 2005)

Thousands of dollars for each minute of down-time

- (e.g., Denver Airport Luggage Handling System)

Huge losses of monetary and intellectual investment

- Rocket boost failure (e.g., Ariane 5)

Business failures associated with buggy software

- (e.g., Ashton-Tate dBase, Ethereum DAO, CrowdStrike outage 2024)

# Failing Software Costs Lives

Potential problems are obvious:

- Software used to control nuclear power plants
- Air-traffic control systems
- Spacecraft launch vehicle control
- Embedded software in cars

A well-known and tragic example: Therac-25 X-ray machine failures

<https://en.wikipedia.org/wiki/Therac-25>

# The Peculiarity of Software Systems

Software seems particularly prone to **faults**

Tiny faults can have catastrophic consequences

- Ariane 5
- Mars Climate Orbiter, Mars Sojourner
- Pentium-Bug
- ...

Rare bugs can occur

- avg. lifetime of a passenger plane: 30 years
- avg. lifetime of a car: < 10 years, but > 1.4B cars in 2022

Logic and implementation errors represent security exploits

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Logic and implementation errors represent **security exploits**

- Meltdown, Spectre,
- (too many others to mention)

# Observation

## Building software is what most of you will do after graduation

- You'll be developing systems in the context above
- Given the increasing importance of software,
  - you may be liable for errors
  - your job may depend on your ability to produce reliable systems

What are the challenges in building reliable and secure software?

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What are the challenges in building reliable and secure software?



# Achieving Reliability in Engineering

## Some well-known strategies from civil/mechanical engineering:

- Precise calculations/estimations of forces, stress, etc.
- Hardware redundancy (“make it a bit stronger than necessary”)
- Robust design (single fault not catastrophic)
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- Software systems compute **non-continuous** functions  
Single bit-flip may change behaviour completely
- Redundancy as replication doesn't help against logical errors  
Redundant SW development only viable in extreme cases
- No physical or modal separation of subsystems  
Local failures often affect whole system
- Software designs have very high logic complexity
- Most SW engineers are untrained in correctness
- Cost efficiency more important than reliability
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# How to Ensure Software Correctness?

A Central Strategy: **Testing**

(others: SW processes, reviews, libraries, ...)

## **Testing against inherent SW errors (“bugs”)**

1. Design test configurations that hopefully are representative
2. Check that the system behaves as intended on them

## Testing against external faults

1. Inject faults (memory, communication) by simulation or radiation
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Exhaustive testing viable only for trivial systems

*Representativeness of test cases/injected faults is subjective*

*How to test for the unexpected? Rare cases?*

*Testing is labor intensive, hence expensive*

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# Complementing Testing: Formal Verification

A Sorting Program:

```
int* sort(int* a) {  
    ...  
}
```

Testing sort:

- $\text{sort}(\{3, 2, 5\}) == \{2, 3, 5\}$  ✓
- $\text{sort}(\{\}) == \{\}$  ✓
- $\text{sort}(\{17\}) == \{17\}$  ✓

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Typically missed test cases

- `sort({2, 1, 2}) == {1, 2, 2}` ✗
- `sort(null) == exception` ✗
- `isPermutation(sort(a), a)` ✗

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# Formal Verification as Theorem Proving

## Theorem (Correctness of sort)

For **any** given non-null int array  $a$ , calling the program  $\text{sort}(a)$  returns an int array that is sorted wrt  $\leq$  *and is a permutation of  $a$ .*

However, methodology differs from mathematics:

1. Formalise the expected property in a logical language
2. Prove the property with the help of an (semi-)automated tool

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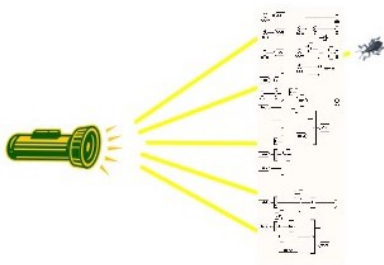
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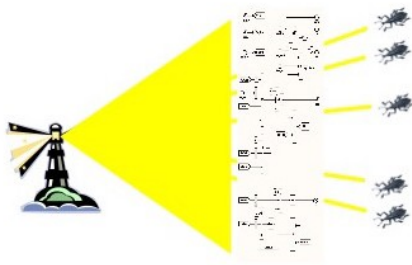
# Contrasting Testing with Formal Verification

*Testing Checks Only the Values We Select*

*Formal Verification Checks Every Possible Value!*



*Even Small Systems Have Trillions  
(of Trillions) of Possible Tests!*



*Finds every exception to the  
property being checked!*

## Formal Methods

A suite of methods and techniques for producing **provably correct** programs by employing a mix of **algorithmic** and **deductive** logical reasoning.

- A formal *specification* capturing the *intended behaviour* of the program is assumed to be provided by a human developer.
- The program is then *checked* against the formal specification, and if it is *proved to satisfy the ascribed specification*, it is deemed “*correct*”.

# Formal Methods

**Rigorous** techniques and tools for the **development and analysis** of computational (hardware/software) systems

- Applied at various stages of the development cycle
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# Main Artefacts in Formal Methods

1. System requirements
2. System implementation

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- a. some formal specification of (1)
- b. some formal execution model of (2)

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Example:

Specifying a Compiler

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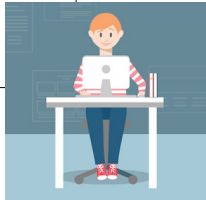
## Program in C

```
#include <stdio.h>

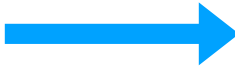
#define IN 1 /* inside a word */
#define OUT 0 /* outside a word */

/* count lines, words, and characters in input */
main()
{
    int c, nl, nw, nc, state;

    state = OUT;
    nl = nw = nc = 0;
    while ((c = getchar()) != EOF) {
        ++nc;
        if (c == '\n')
            ++nl;
        if (c == ' ' || c == '\n' || c == '\t')
            state = OUT;
        else if (state == OUT) {
            state = IN;
            ++nw;
        }
    }
    printf("%d %d %d\n", nl, nw, nc);
}
```



*compile*



## Program in Arm Assembly

```
792415C0    55          push ebp
792415C1    89E5        mov ebp, esp
792415C3    8B45 08     mov eax, [ebp+0x08]
792415C6    DB28       fld tword [eax]
792415C8    8B4D 0C     mov ecx, [ebp+0x0C]
792415CB    DB29       fld tword [ecx]
792415CD    DEC1       faddp
792415CF    8B55 10     mov edx, [ebp+0x10]
792415D2    DB3A       fstp tword [edx]
792415D4    DB68 0A     fld tword [eax+0x0A]
792415D7    DB69 0A     fld tword [ecx+0x0A]
792415DA    DEC1       faddp
792415DC    DB7A 0A     fstp tword [edx+0x0A]
792415DE    55          pop ebp
792415DF    0000       ret 0x000C
```



## Program P in C

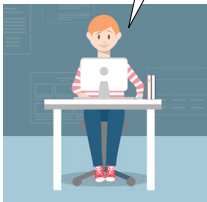
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        }
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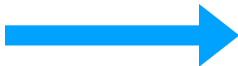
*interpret-as-C*



$$\text{Result}(P, \text{input}) = R_c = R_{\text{arm}} = \text{Result}(\text{compile}(P), \text{input})$$



*compile*



## Program *compile*(P) in Arm Assembly

```

792415C0 55      push ebp
792415C1 89E5    mov ebp, esp
792415C3 8B45 08 mov eax, [ebp+0x08]
792415C6 DB28    fld tword [eax]
792415C8 8B4D 0C mov ecx, [ebp+0x0C]
792415CB DB29    fld tword [ecx]
792415CD DEC1    faddp
792415CF 8B55 10 mov edx, [ebp+0x10]
792415D2 DB3A    fstp tword [edx]
792415D4 DB68 0A fld tword [eax+0x0A]
792415D7 DB69 0A fld tword [ecx+0x0A]
792415DA DEC1    faddp
792415DC DB7A 0A fstp tword [edx+0x0A]
792415DF 5D      pop ebp
792415E0 C2 0C00 ret 0x000C
    
```

*interpret-as-x86*



## Compiler Specification:

For *any* program  $P$ , and *any* input, the result of *interpreting*  $P$  with input in  $\mathbf{C}$  is the same as the result of *executing compilation* of  $P$  with input in **Arm Assembly**.

or, equivalently

## Correctness Theorem:

$$\forall P, \text{input}, \textit{interpret}_{\mathbf{C}}(P, \text{input}) = \textit{execute}_{\text{arm}}(\textit{compile}(P, \text{input}))$$

## Correctness Theorem:

$$\forall P, \text{input}, \textit{interpret}_c(P, \text{input}) = \textit{execute}_{\text{arm}}(\textit{compile}(P, \text{input}))$$

**Proof:** ???

## Assumptions:

- Meaningful definition of *interpret*<sub>C</sub> is given and fixed
- Meaningful definition of *execute*<sub>arm</sub> is given and fixed
- Specific implementation of *compile* is given and fixed
- Considered programs P is are valid and written in C

must be trusted  
(i.e., better be “sane”)

## Correctness Theorem:

$$\forall P, \text{in}, \textit{interpret}_C(P, \text{in}) = \textit{execute}_{\text{arm}}(\textit{compile}(P, \text{in}))$$

**Proof:** ???

once proven,  
does not have  
to be trusted

# Why Use Formal Methods

1. **Contribute to the overall quality** of the final product thanks to mathematical modelling and formal analysis
2. **Increase confidence** in the correctness/robustness/security of a system
3. **Find more flaws** and **earlier** (i.e., during specification and design vs. testing and maintenance)



# Formal Methods: The Vision

- **Complement** other analysis and design methods
- Help **find bugs** in code **and** specification
- **Reduce** development, and testing, **cost**
- **Ensure** certain **properties** of the formal system model
- Should be highly **automated** (perhaps with AI in the future)

# A Warning

- The effectiveness of FMs is still debated
- There are persistent myths about their practicality and cost
- FMs are not yet as widespread in industry as they could be
- They are mostly used in the development of safety-, business-, or mission-critical software, where the cost of faults is high

🏠 SPLASH 2024 (series) / 🗂️ Keynotes /


## Trillions of Formally Verified Authorizations a day!

**Track** [SPLASH 2024 Keynotes](#)

**When** [Fri 25 Oct 2024 09:00 - 10:30 at IBR Ballroom - Keynote - Neha Rungta](#)

**Abstract** To control access to their data and resources, AWS users write policies that express fine-grained permissions. An authorization engine evaluates these policies trillions of times a day to determine if access is allowed. The authorization engine is a critical part of the security, availability, and correctness of AWS. To raise the bar in the security and correctness of this engine, we replaced it with a formally verified one. It was absolutely critical that users were not impacted by the change. Over a period of months and over quadrillion tests, we gathered data on the impact, and deployed the new verified engine. And then what happens? Come to the talk to find out.

**Session Program** [Fri 25 Oct](#)  
Displayed time zone: Pacific Time (US & Canada) [change](#)



**Neha Rungta**  
Amazon Web Services

# The Main Point of Formal Methods is **Not**

- To show “correctness” of entire systems
  - What **is** correctness? Go for specific properties!
- To replace testing entirely
  - FMs typically do not go below byte code level
  - Some properties are not (easily) formalisable
- To replace good design practices

There is no silver bullet!

No correct system w/o clear requirements & good design

# Overall Benefits of Using Formal Methods

1. Forces developers to think *systematically* about issues
2. Improves the quality of specifications, *even without formal verification*
3. Leads to *better design*
4. Provides a *precise reference* to check requirements against
5. Provides *rigorous documentation* within a team of developers
6. Gives *direction* to later development phases
7. Provides a basis for *reuse* via specification matching
8. Can replace (*infinitely*) *many* test cases
9. Facilitates automatic *test case generation*

# Specifications: What the system **should** do

- Individual properties
  - Safety properties: **something bad will *never* happen**
  - Liveness properties: **something good will happen *eventually***
  - Non-functional properties: runtime, memory, usability, . . .
- “Complete” behaviour specification
  - Equivalence check
  - Refinement
  - Data consistency
  - . . .

## Formal Specification

*The expression in some **formal language** and  
at some level of **abstraction**  
of a collection of **properties** that some system should **satisfy***

*[Axel van Lamsweerde]*

# Formal Specification

The expression in some *formal language* and at some level of *abstraction* of a collection of *properties* that some system should *satisfy* [van Lamsweerde]

formal language:

- syntax can be mechanically processed and checked
- semantics is defined unambiguously by mathematical means

abstraction:

- above the level of source code
- several levels possible

properties:

- expressed in some formal logic
- have a well-defined semantics

satisfaction:

- ideally (but not always) decided mechanically
- based on automated deduction and/or model checking techniques

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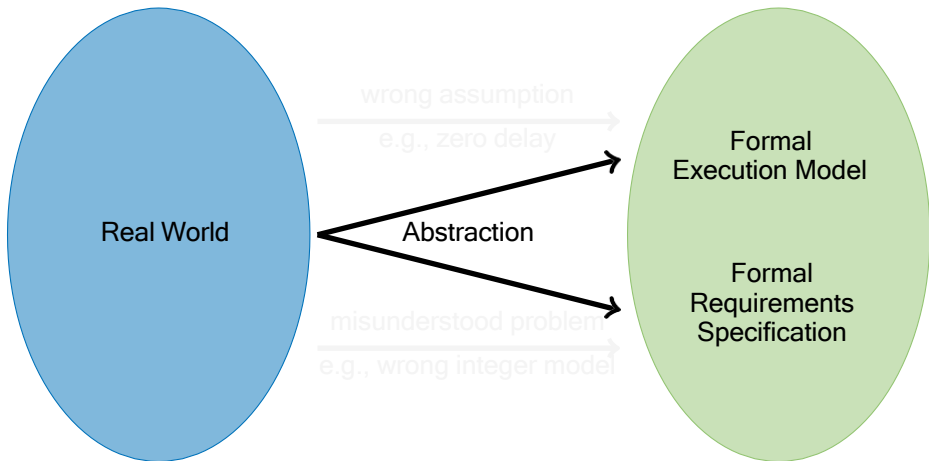
# Formalization Helps to Find Bugs in Specs!

- *Well-formedness* and *consistency* of formal specs are machine-checkable
- Fixed *signature* (set of behaviours) helps spot incomplete specs
- Failed verification of *implementation against specs* provides feedback on errors
  - in the implementation or
  - in the (formalisation of the) spec

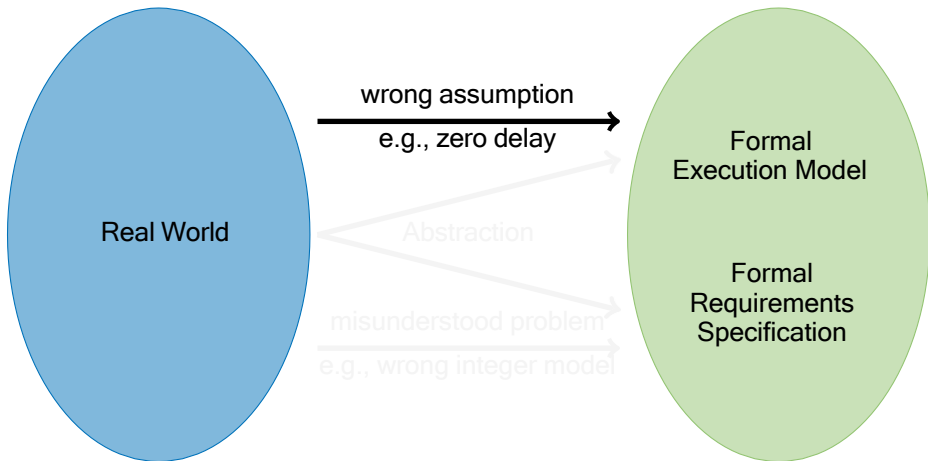
# A Fundamental Fact

Formalizing system requirements is hard

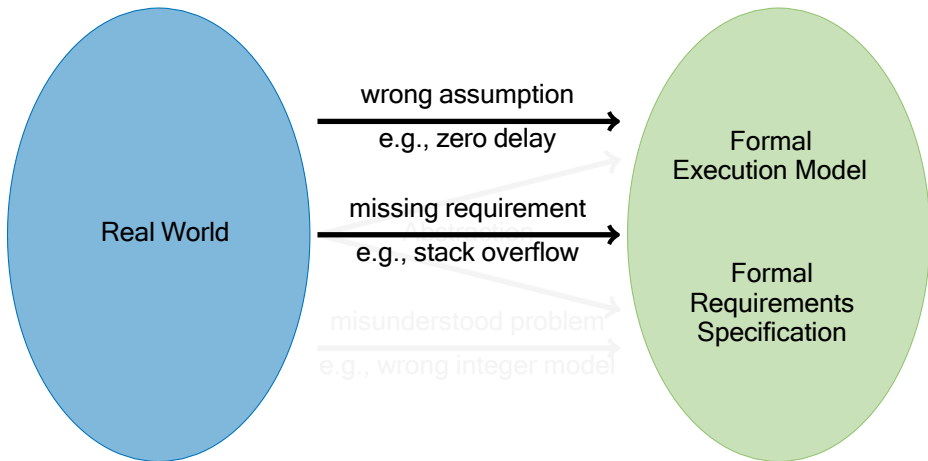
# Difficulties in Creating Formal Models



# Difficulties in Creating Formal Models

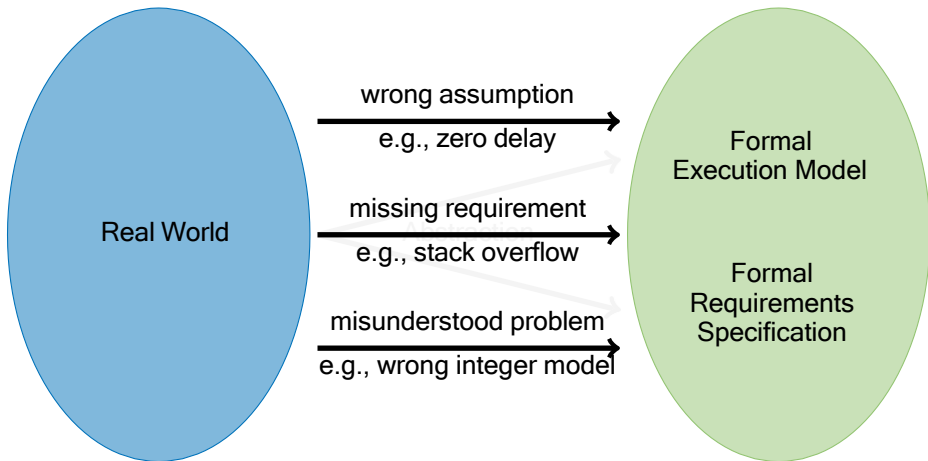


# Difficulties in Creating Formal Models





# Difficulties in Creating Formal Models



## Another Fundamental Fact

Proving properties of systems can be hard

# Level of System Description

## High level (modelling)

- Abstract clean semantics
- Easier to program
- Automatic proofs (sometimes) are possible

:

## Low level (implementation level)

- Realistic programming language
- Often can be directly executed
- Automatic proofs are (mostly) impossible

Alloy



## Summary So Far

- Software is becoming pervasive and very complex
- Current development techniques are inadequate
- Formal methods . . .
  - are not a panacea, but will be increasingly necessary
  - are (more and more) used in practice
  - can shorten development time
  - can push the limits of feasible complexity
  - can increase product quality
  - can improve system security
- We will learn to use several different formal methods, for different development stages

Next: formal methods in action!