Зависимые Типы для верификации Реалистичного Кода

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Тип — это *множество* данных и *операций* над ними.

Как *описать* вычисления

- Формальная спецификация
- Суперкомпиляция
- Равенство
- Взаимозаменяемость
- Завершаемость
- Non-interference

Как *реализовать* вычисления

- Указатели
- **while (true)** {…}
- Input/Output
- **goto/break/continue**
- Exceptions
- Многопоточность
- Распределенные вычисления

λ calculus
A. Church (1930s)

Simply typed λ calculus
A. Church (1940)

Turing Machine
A. Turing (1936)

Типы как Множества

```
Datatype unit := tt.
```

$$\text{unit} \stackrel{\text{def}}{=} \{ \text{tt} \}$$

$$\text{tt} \in \text{unit}$$

```
Datatype unit := tt.
```

$$\text{unit} \stackrel{\text{def}}{=} \{\ \text{tt}\ \}$$

$$\text{tt} : \text{unit}$$

```
Datatype bool := true | false.
```

$$\text{bool} \stackrel{\text{def}}{=} \{\ \text{true}, \text{false}\ \}$$

true : bool

false : bool

```
Datatype nat := 0 | .+1 of nat.
```

$$\text{nat} \stackrel{\text{def}}{=} \{\, 0,\ \underbrace{(0.\!+\!1)}_{1},\ \underbrace{(0.\!+\!1.\!+\!1)}_{2},\ \ldots \,\}$$

$$0 : \text{nat} \qquad\qquad \frac{n : \text{nat}}{n.\!+\!1 : \text{nat}}$$

```
Function negate : bool -> bool :=
  fun b => match b with
    | true  => false
    | false => true
  end.
```

$$\frac{x : A \ \vdash \ e : B}{\mathbf{fun}\ x \Rightarrow e : A \to B}$$

$$\frac{f : A \to B \qquad x : A}{f(x) : B}$$

```
Datatype Record2 A B := {a : A; b : B}

Datatype Record3 A B C :=
                {a : A; b : B; c : C}
```

$$\frac{}{\texttt{Record3}\ A\ B\ C \ <:\ \texttt{Record2}\ A\ B}$$

$$\frac{P <: P' \qquad e : P' \rightarrow Q' \qquad Q' <: Q}{e : P \rightarrow Q}$$

$$\frac{x : A,\ \ f : A \rightarrow B \ \vdash\ e : B}{(\mathrm{Rec}\ f : A \rightarrow B := \mathrm{fun}\ x \Rightarrow e) : A \rightarrow B}$$

```
Rec even : nat -> bool :=
  fun n => match n with
    | n'.+1 => negate (even n')
    | 0     => true
  end.
```

```
Rec even : nat -> bool :=
  fun n => match n with
    | n'.+1 => negate (even n')
    | 0     => 0
  end.
```

Wrong type: bool expected, but nat found.

$$\Pi(x: A).\ B(x)$$

```
F = Π(b: bool). if b then nat else unit
```

```
Function foo : F :=
  fun b => match b with
             | true  => 0
             | false => tt
  end
```

Checkpoint 1
Типы как Множества

- Программы — значения, элементы множеств;
- *Well-typed program don’t go wrong* (R. Milner);
- Типы — спецификация программ;
- Проверка типов — верификация программ;
- Модульность по принципу подстановки: *тип программы независим от контекста ее применения*.

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```
x := 0;

Function x_non_neg : bool -> bool :=
  fun b => match b with
    | true  => x >= 0;
    | false => x < 0;
  end.

x := 1;
x_non_neg(true); // true

x := -1;
x_non_neg(true); // false
```

```
x := 0;

Function x_non_neg: Πb. if b then nat else unit :=
   fun b => match b with
     | true  => if x >= 0 then 0 else tt;
     | false => tt;
   end.

x := 1;
x_non_neg(true); // 0 : nat

x := -1;
x_non_neg(true); // tt : unit
```

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$$\{ P \}\ c\ \{ Q \}$$

предусловие постусловие

$$\underbrace{s_0 : P(s_0) \rightarrow s_1 \rightarrow s_2 \rightarrow \ldots \rightarrow s_n}_{c} \Rightarrow Q(s_n)$$

$$\{\ \text{True}\ \} \quad x := 3 \quad \{\ x = 3\ \}$$

$$\{\, Q[e/x] \,\}\ x := e\ \{\, Q \,\} \qquad \text{(Assign)}$$

$$\{\, 3 = 3 \,\}\ x := 3\ \{\, x = 3 \,\}$$

$$\frac{\{\ P\ \}\ c_1\ \{\ Q\ \} \qquad \{\ Q\ \}\ c_2\ \{\ R\ \}}{\{\ P\ \}\ c_1;\ c_2\ \{\ R\ \}}\ (\text{Seq})$$

$\{???\}$ x := 3; y := x $\{x = 3 \wedge y = 3\}$

$$\frac{\{\ P\ \}\ c_1\ \{\ Q\ \} \qquad \{\ Q\ \}\ c_2\ \{\ R\ \}}{\{\ P\ \}\ c_1;\ c_2\ \{\ R\ \}}\ (\mathrm{Seq})$$

$\boxed{\{3 = 3 \wedge 3 = 3\}}$

x := 3;

$\{x = 3 \wedge x = 3\}$ (Assign)

y := x

$\{x = 3 \wedge y = 3\}$ (Assign)

$$\frac{P \Rightarrow P' \qquad \{\, P' \,\}\ c\ \{\, Q' \,\} \qquad Q' \Rightarrow Q}{\{\, P \,\}\ c\ \{\, Q \,\}}\ (\text{Conseq})$$

$$\{\ \textbf{True}\ \} \quad \Rightarrow \quad \{3 = 3 \wedge 3 = 3\}$$

$$x := 3;\ y := x$$

$$\{x = 3 \wedge y = 3\}$$

$$\frac{P \Rightarrow P' \qquad \{\, P' \,\}\ c\ \{\, Q' \,\} \qquad Q' \Rightarrow Q}{\{\, P \,\}\ c\ \{\, Q \,\}}\ \text{(Conseq)}$$

$$\{\, \textbf{True} \,\} \quad x := 3;\ y := x \ \ \{x = 3 \land y = 3\}$$

$$\frac{P \Rightarrow P' \qquad \{\, P' \,\}\ c\ \{\, Q' \,\} \qquad Q' \Rightarrow Q}{\{\, P \,\}\ c\ \{\, Q \,\}}\ \text{(Conseq)}$$

$$\frac{P <: P' \qquad e : P' \rightarrow Q' \qquad Q' <: Q}{e : P \rightarrow Q}$$

$$\forall\, a, b,$$

$$\Big(\{x = a \wedge y = b\}\ t := x;\ \ x := y;\ \ y := t\ \{x = b \wedge y = a\} \Big)$$

```
Π(b: bool). if b then nat else unit
```

$$\frac{\{\boxed{Inv} \wedge b\}\ c\ \{\boxed{Inv}\}}{\{\boxed{Inv}\}\ \textbf{while}\ b\ \textbf{do}\ c\ \{\boxed{Inv} \wedge \neg b\}}\ (\text{While})$$

$$\frac{x : A,\ \boxed{f : A \to B} \vdash e : B}{(\textbf{Rec}\ f : A \to B := \textbf{fun}\ x \Rightarrow e) : \boxed{A \to B}}$$

Почему Hoare Logic не работает

```
int ival = 3;
int *x = …;
int *y = …;
```

$\{ x \mapsto - \wedge y \mapsto b \}$ `*x = &ival;` $\{ x \mapsto 3 \wedge y \mapsto b \}$

Что делать, если `x` и `y` указывают на одно и то же значение?

```
int ival = 3;
int *x = …;
int *y = …;
```

$\{\ x \mapsto -\ \wedge\ y \mapsto b\ \}$

```
x = &ival;
```

$\{\ x \mapsto \texttt{ival}\ \wedge$

$(x \neq y \wedge y \mapsto b) \vee (x = y \wedge y \mapsto \texttt{ival})\}$

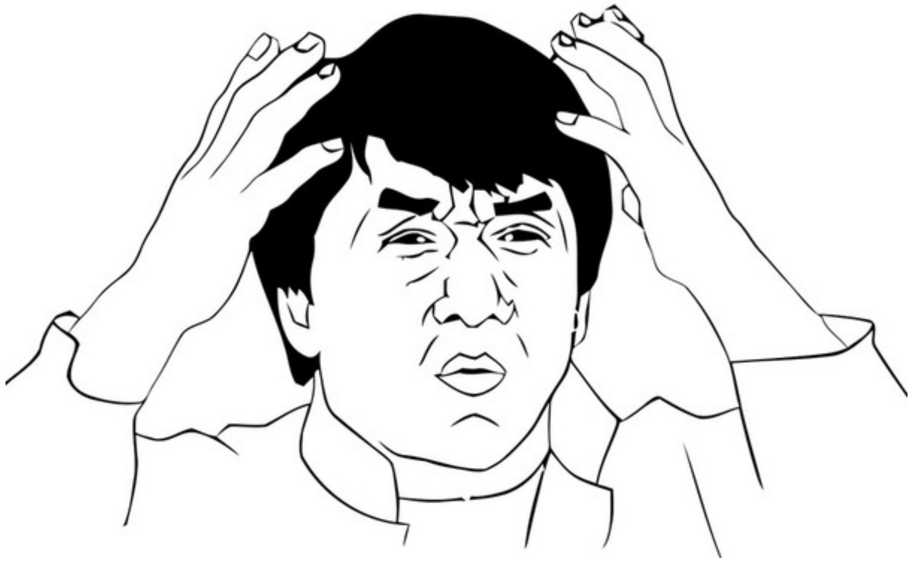
```
int ival = 3;
int *x = …;
int *y = …;
int *z = …;
```

$\{\ x \mapsto - \wedge y \mapsto b\ \}$

```
x = &ival;
```

$\{\ x \mapsto \texttt{ival} \wedge$

$(x \neq y \wedge y \mapsto b) \vee (x = y \wedge y \mapsto \texttt{ival})\}$



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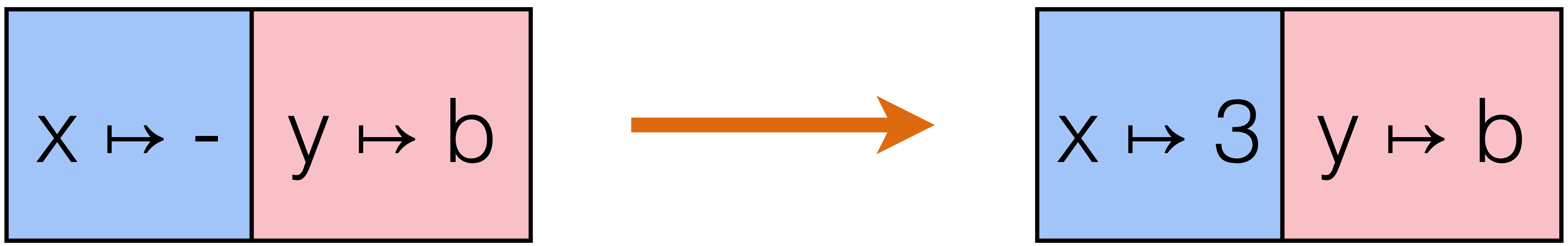
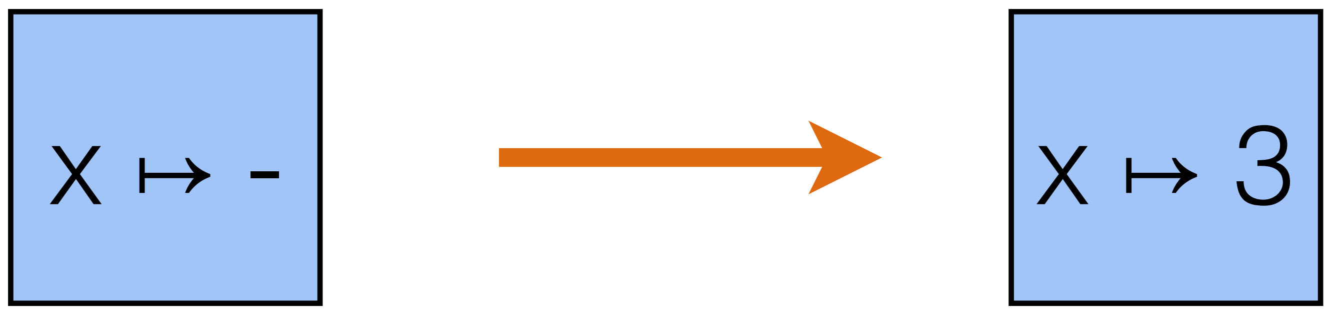
Separation Logic
J. Reynolds (2002)

$\{\, x \mapsto - \wedge y \mapsto b \,\}$ `*x = 3;` $\{\, x \mapsto 3 \wedge y \mapsto b \,\}$



$\{\ x \mapsto - \uplus y \mapsto b\ \}$ `*x = 3;` $\{\ x \mapsto 3 \uplus y \mapsto b\ \}$

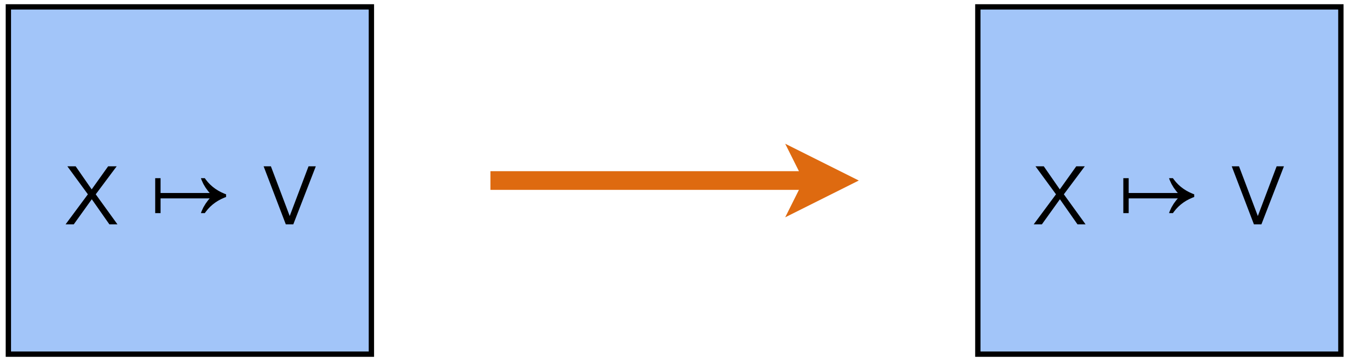
несвязное объединение регионов памяти



$\{\ x \mapsto -\ \}$ `*x = 3;` $\{\ x \mapsto 3\ \}$

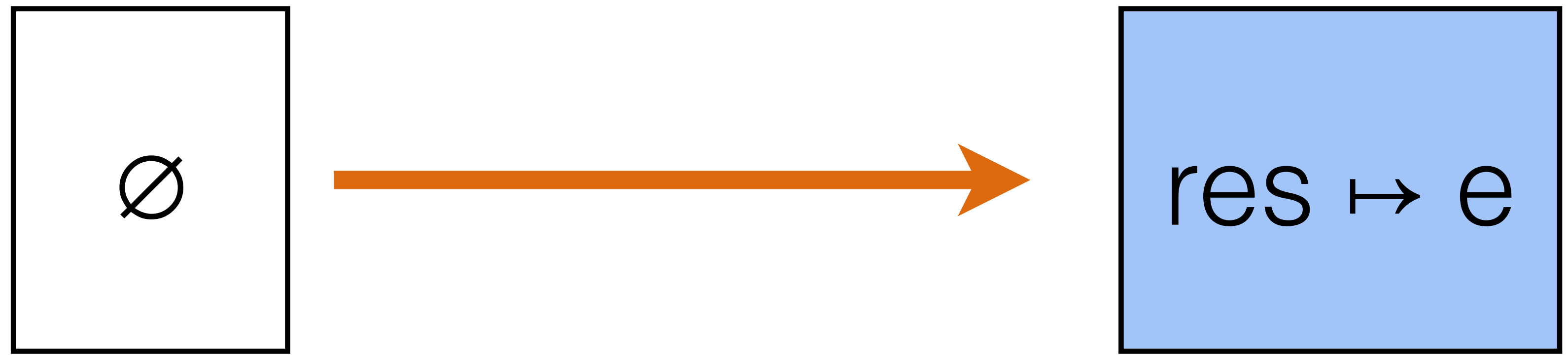
$\{h \mid h = x \mapsto -\}$ $*x = e;$ $\{res, h \mid h = x \mapsto e \wedge res = tt\}$

(Write)



$\{h \mid h = x \mapsto v\}$ *x $\{res, h \mid h = x \mapsto v \wedge res = v\}$

(Read)

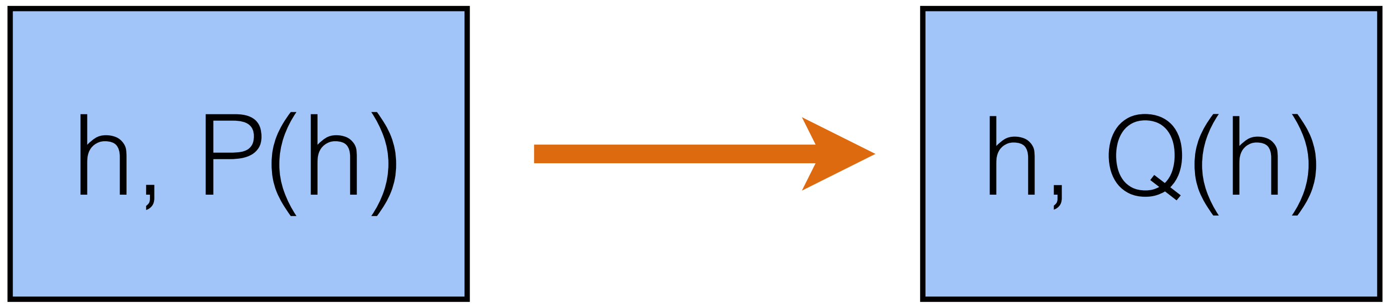


$\{h \mid h = \emptyset\}$ alloc(e); $\{res, h \mid h = res \mapsto e\}$

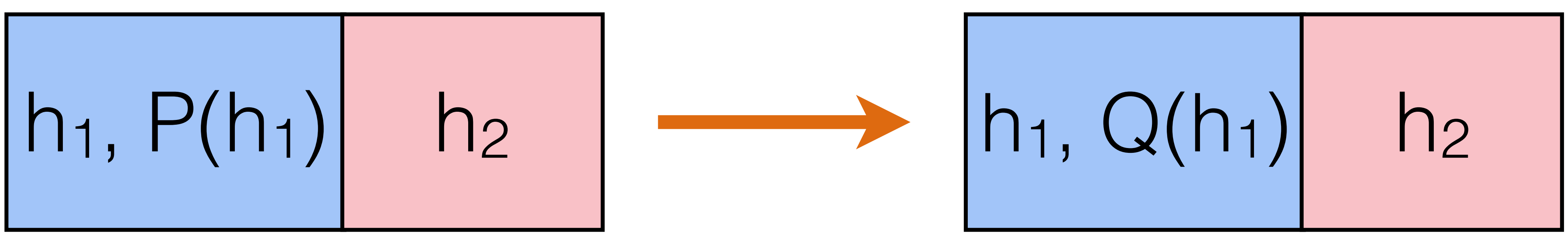
(Alloc)

$\{h \mid h = x \mapsto -\}$ free(x); $\{res, h \mid h = \emptyset \wedge res = tt\}$

(Free)

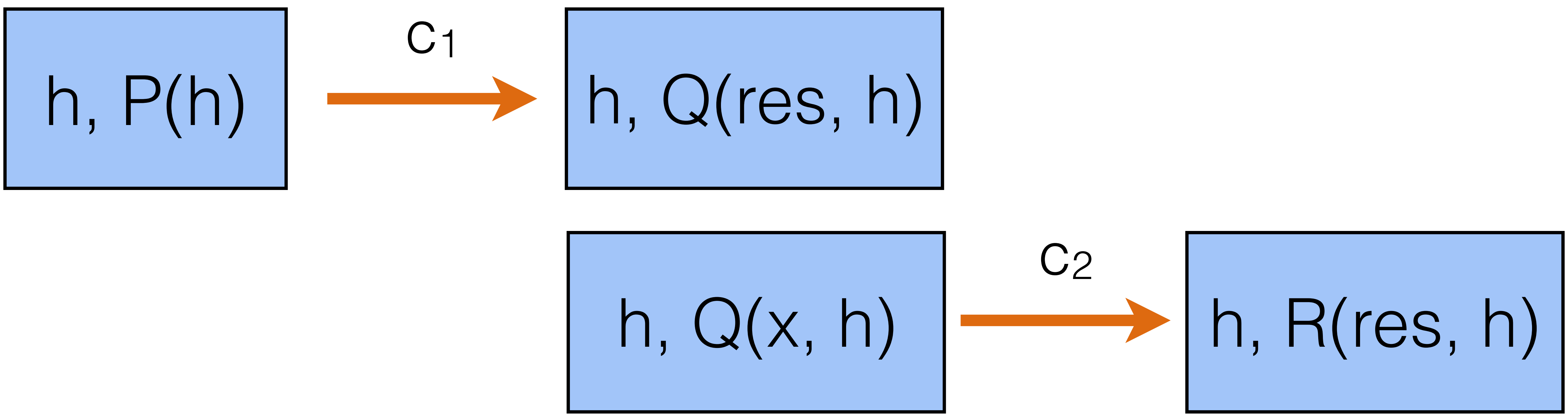


$$\frac{\{h \mid \mathbf{P}(h)\}\ c\ \{res, h \mid \mathbf{Q}(h)\}}{\{h \mid \exists h_1,\ \ h = h_1 \uplus h_2 \wedge\ \mathbf{P}(h_1)\}\ c\ \{res, h \mid \exists h_1,\ \ h = h_1 \uplus h_2 \wedge\ \mathbf{Q}(h_1)\}}\ \text{(Frame)}$$

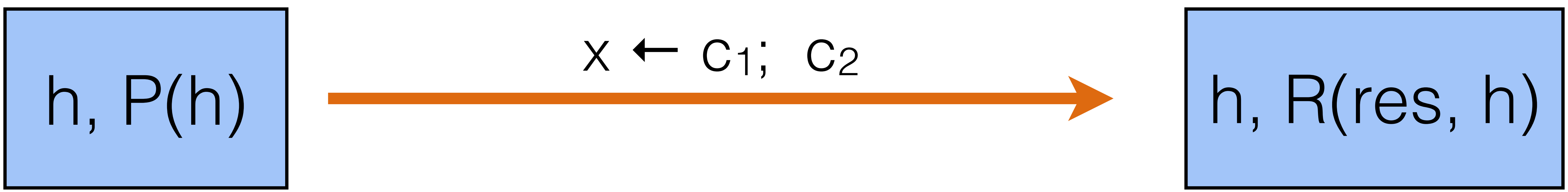


$\{h \mid \mathbf{P}(h)\}\ \ \mathbf{ret}\ e;\ \{res, h \mid \mathbf{P}(h) \land res = e\}$ (Return)

$$\frac{\begin{array}{c}\{h \mid \mathbf{P}(h)\}\ c_1\ \{res, h \mid \mathbf{Q}(res, h)\} \\ \{h \mid \mathbf{Q}(x, h)\}\ c_2\ \{res, h \mid \mathbf{R}(res, h)\}\end{array}}{\{h \mid \mathbf{P}(h)\}\ \ x \leftarrow c_1;\ \ c_2\ \{res, h \mid \mathbf{R}(res, h)\}}\ \text{(Bind)}$$



$$\dfrac{\begin{array}{c}\{h \mid \mathbf{P}(h)\}\ c_1\ \{res, h \mid \mathbf{Q}(res, h)\} \\ \{h \mid \mathbf{Q}(x, h)\}\ c_2\ \{res, h \mid \mathbf{R}(res, h)\}\end{array}}{\{h \mid \mathbf{P}(h)\}\ \ x \leftarrow c_1;\ \ c_2\ \{res, h \mid \mathbf{R}(res, h)\}}\ \text{(Bind)}$$



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J. Reynolds (2002)

Facebook Infer
P.O'Hearn et al. (2013)

SL Спецификации как Типы

- Программы — комбинации операций с памятью;
- “Типы” — пред-/постусловия;
- Принцип подстановки (Frame rule);
- *Well-typed programs don’t go wrong.*

Hoare/Separation Logic
+
Dependent Types

Hoare Types

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Separation Logic
J. Reynolds (2002)

Hoare Type Theory
A. Nanevski (2006)

Facebook Infer
P.O'Hearn et al. (2013)

$$\forall\, x_1, x_2, \ldots \; \{\, h \mid P(h, x_i) \,\} \; c \; \{ res, h \mid Q(res, h, x_i) \,\}$$

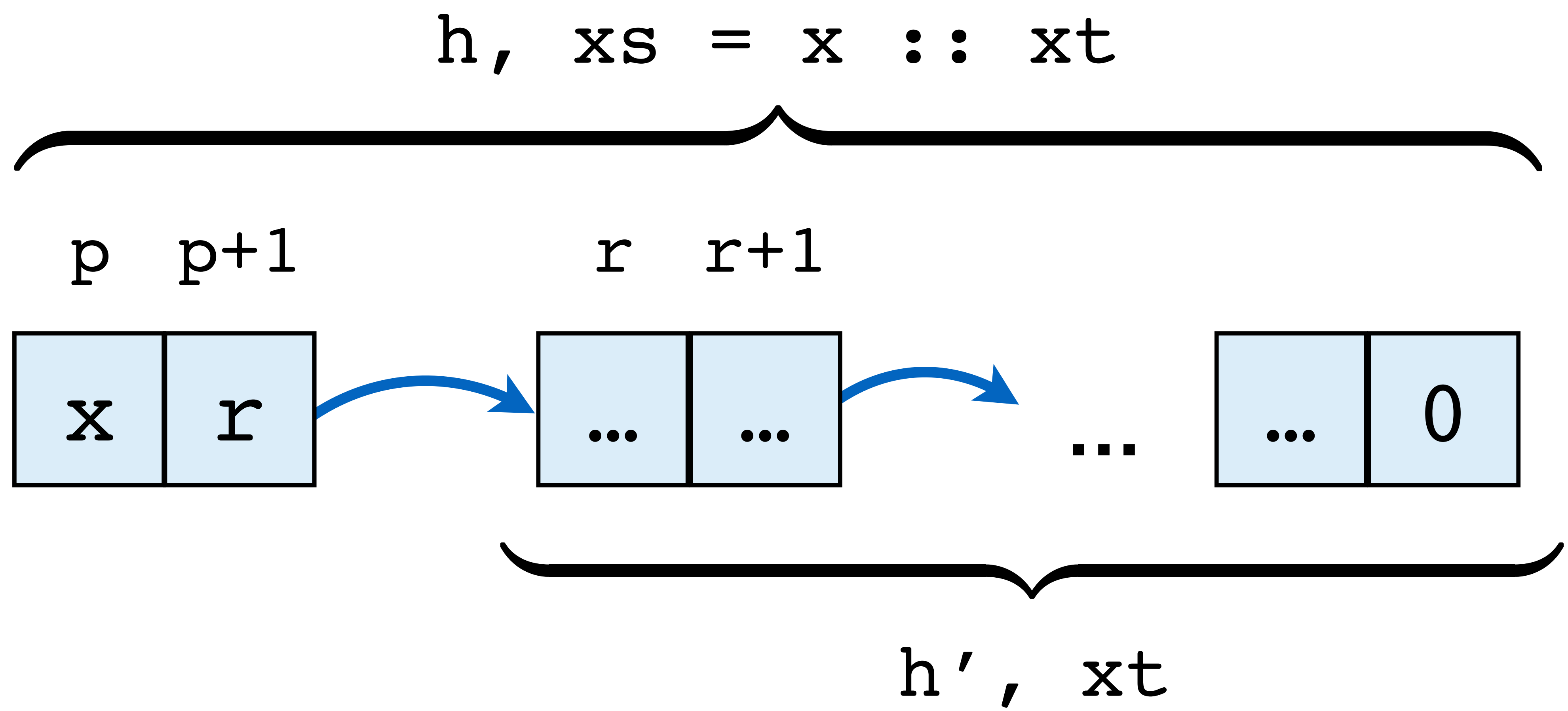
$$c : \{x_1, x_2, \ldots\}\ HT\ (P, Q)$$

```
alloc : Π (A : Type) (v : A),
          HT (fun h ⇒ h = ∅,
              fun res h ⇒ h = res ↦ v)
```

$\boxed{\varnothing}$

$\boxed{res \mapsto v}$

Манипуляции со связными списками




```
Fixpoint llist (p : ptr)(xs : list T) (h: heap) :=
  if xs is x :: xt then
    ∃ r h',
        h = p ↦ x ⊎ (p+1 ↦ r) ⊎ h' ∧ llist r xt h'
  else p = null ∧ h = ∅ .
```

```
Program Definition llist_map f
:= Fix (fun lmap p =>
     Do (if p == null
         then return tt
         else t ← *p;
              *p = f(t);
              nxt ← *(p + 1);
              lmap nxt)).
```

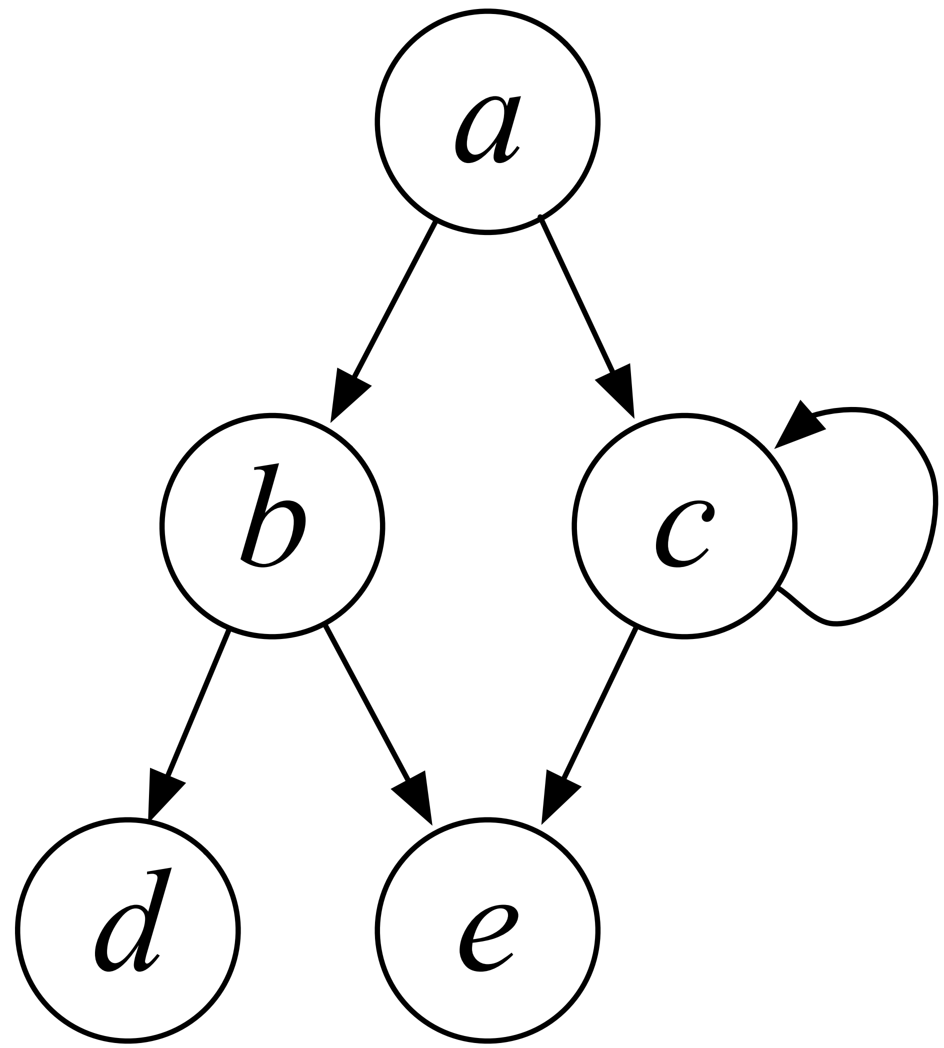
```
Definition llist_map_type (f : T -> S) :=
  forall (p : ptr),
    {xs : list T},
    HT (fun h => llist p xs h,
        fun (_ : unit) h =>
          llist p (map f xs) h).


Program Definition llist_map f : llist_map_type f
:= Fix (fun lmap p =>
    Do (if p == null
        then return tt
        else t ← *p;
             *p = f(t);
             nxt ← *(p + 1);
             lmap nxt)).
```

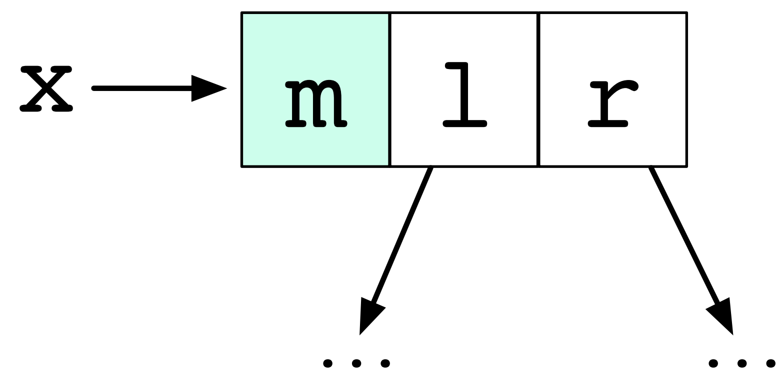
Proof. (6 LOC) **Qed.**

Checkpoint 2: Зависимые типы для программ указателями

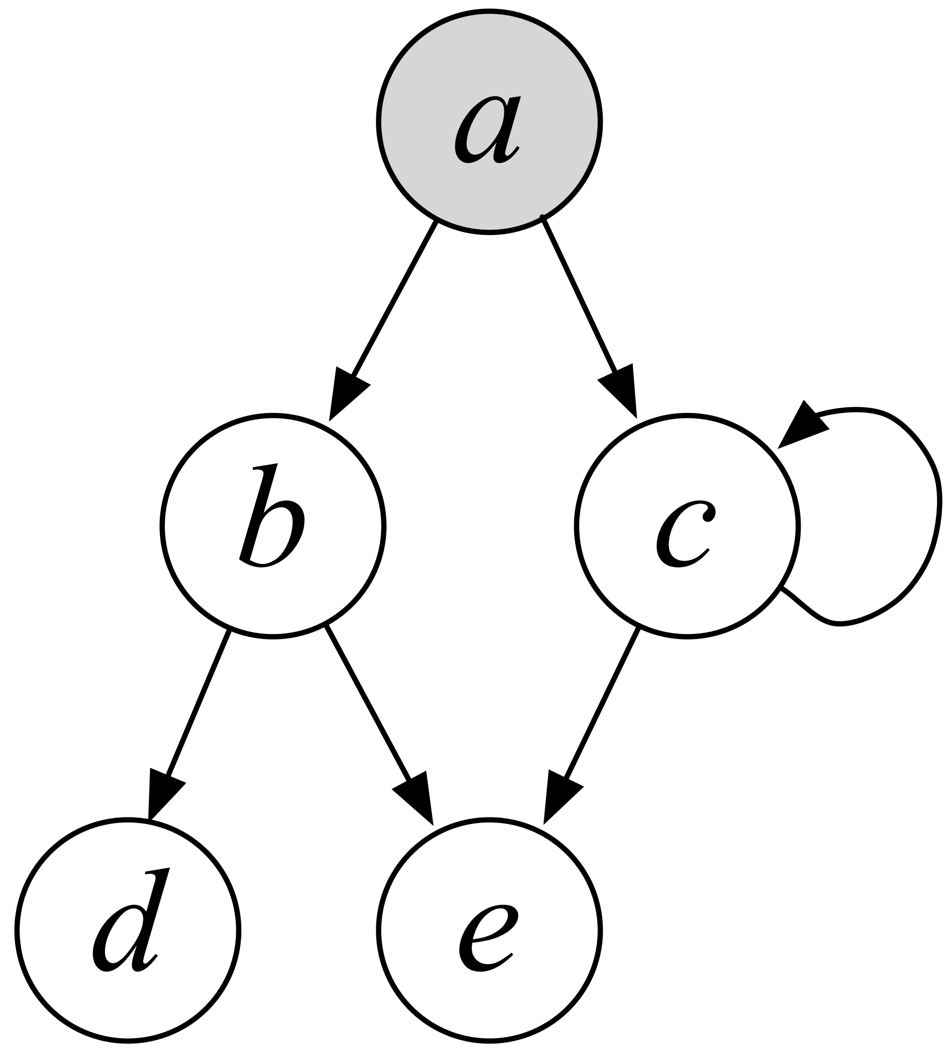
- Зависимые типы описывают *состояние памяти*;
- *Правила вывода* Hoare Types соответствуют *доказательствам* в Separation Logic;
- Проверка типов — верификация программ;
- *Well-typed programs don't go wrong* (again…).

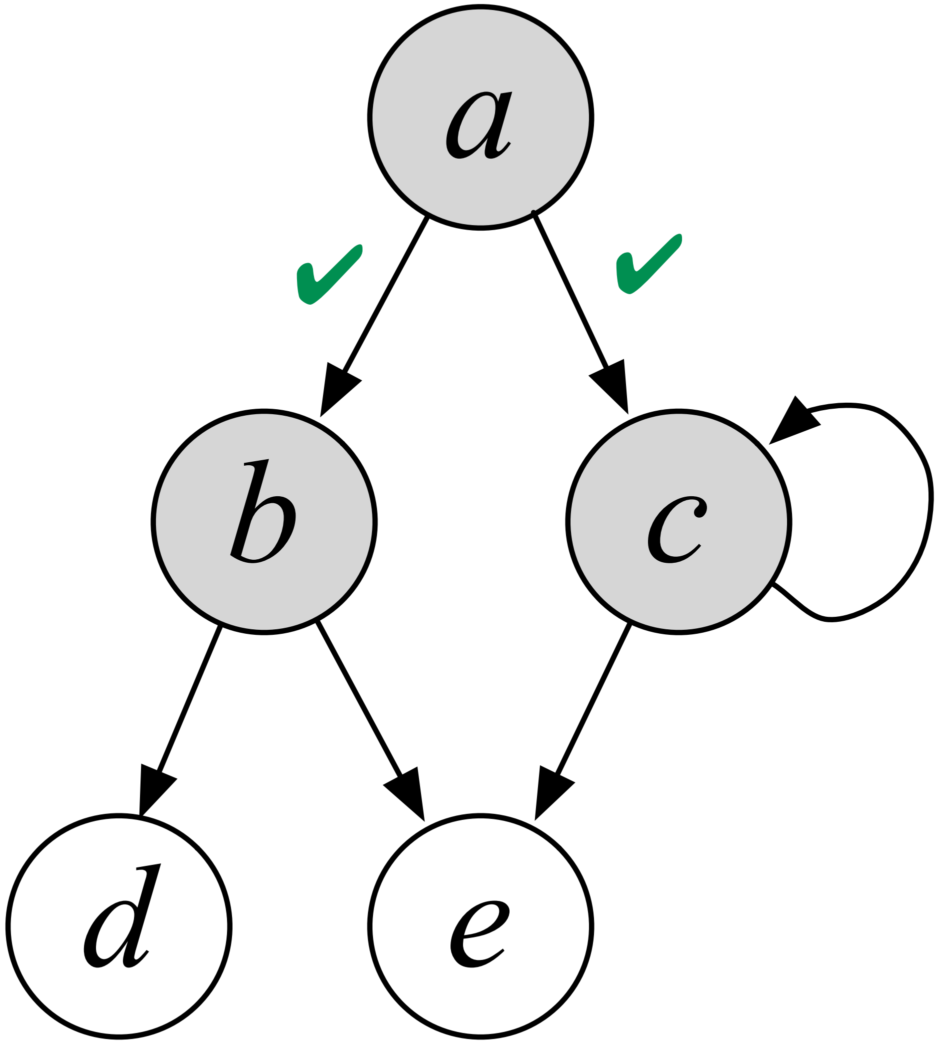
Многопоточность


a
b
c
d
e




```
Program Definition span (x : ptr) : bool = {
  if x == null then return false;
  else
    b ← CAS(x->m, 0, 1);
    if b then
      (rₗ,rᵣ) ← (span(x->l) || span(x->r));
      if ¬rₗ then x->l := null;
      if ¬rᵣ then x->r := null;
      return true;
    else return false;
}
```


a
b
c
d
e


a
b
c
d
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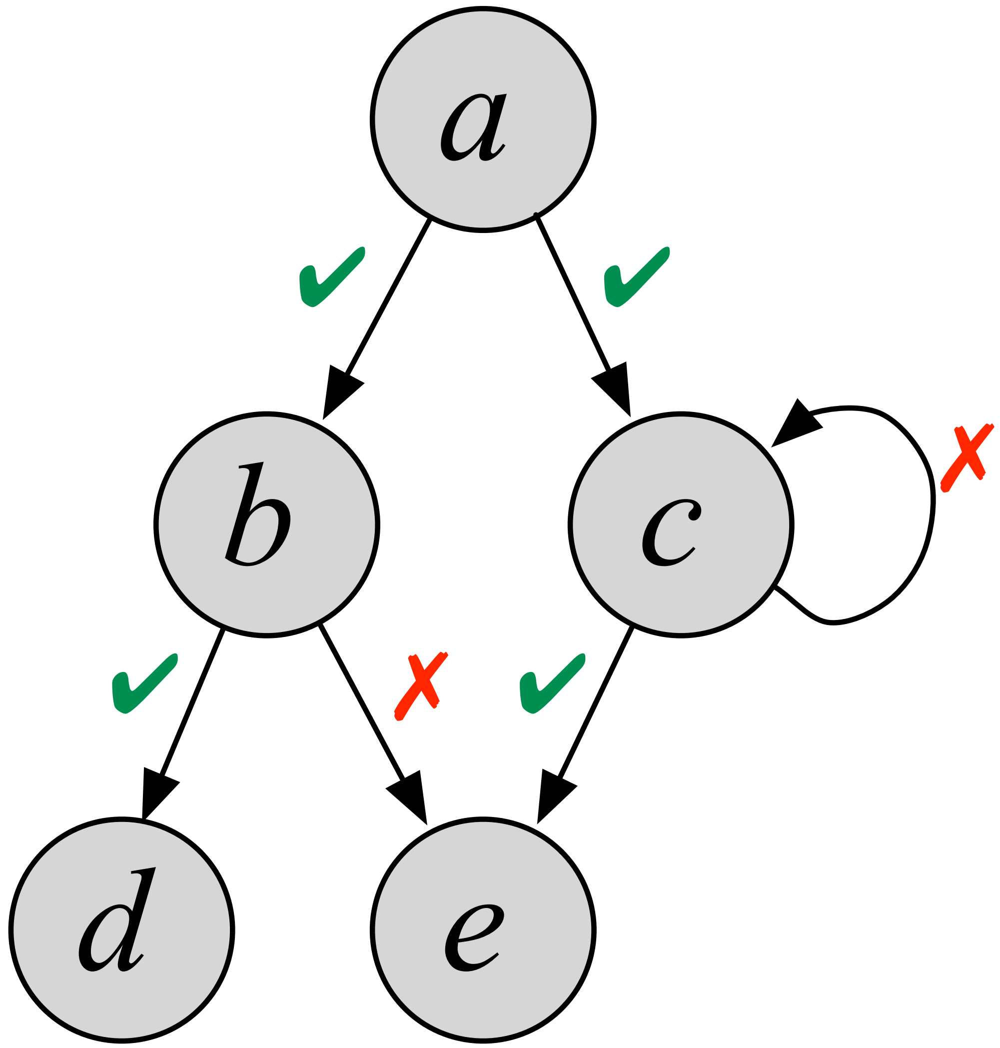
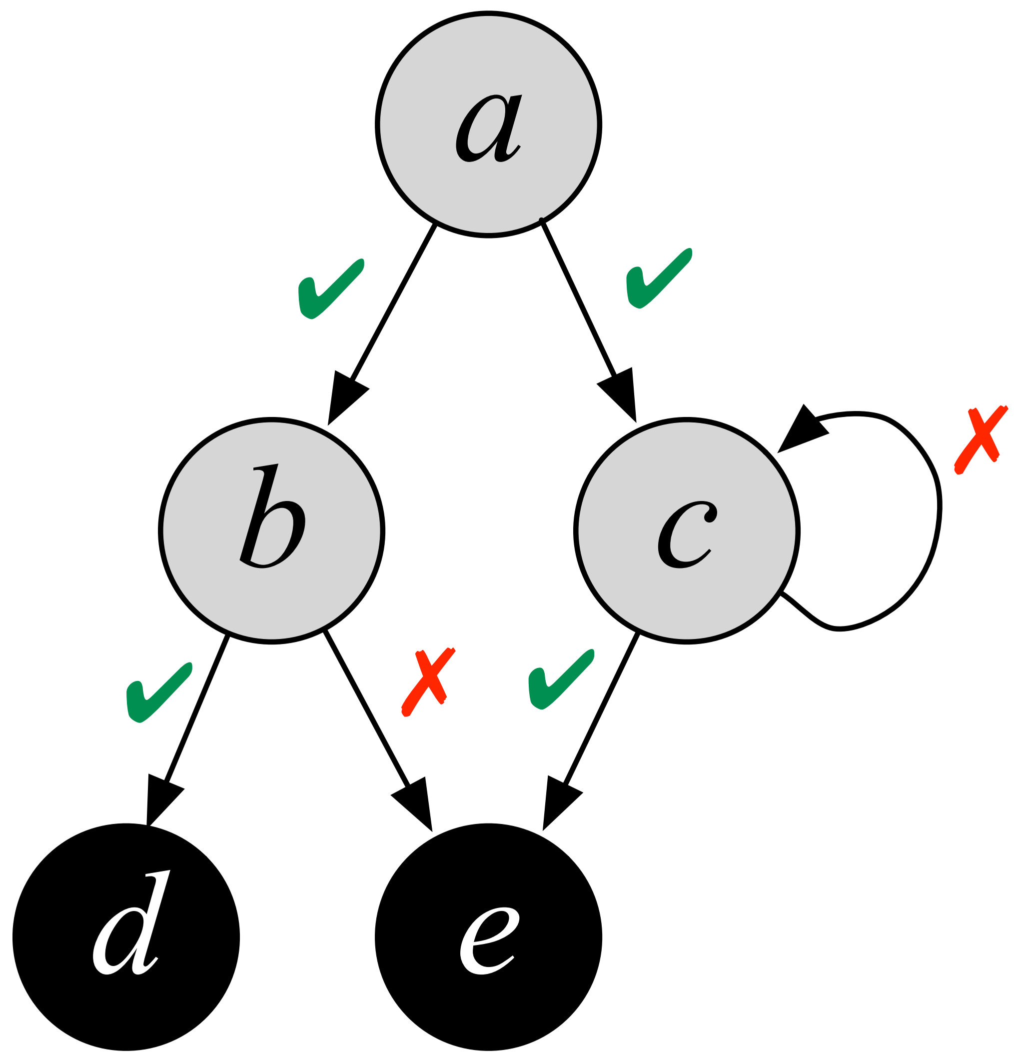
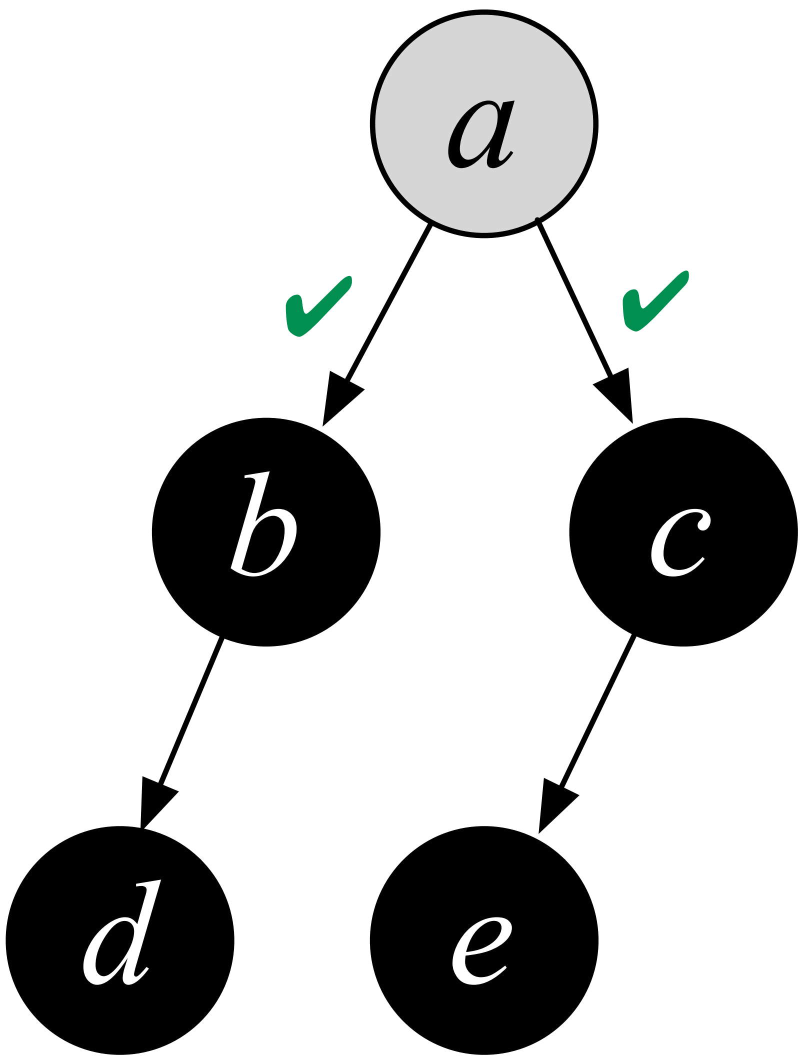
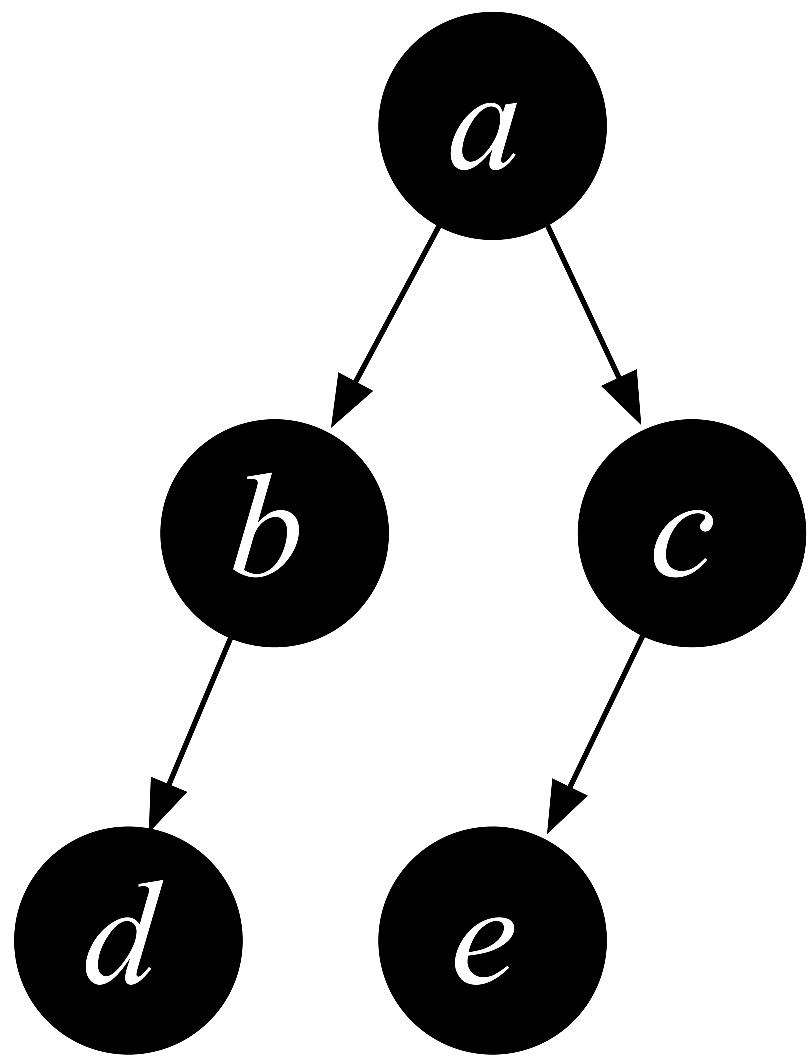

a
b
c
d
e


a
b
c
d
e
✔
✔
✔
✗
✔
✗


a
b
c
d
e

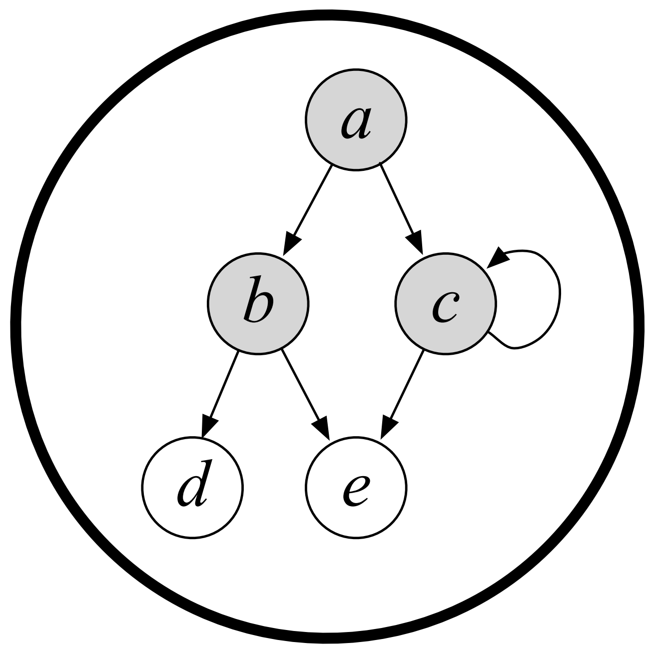

a
b
c
d
e

Верификация span

```
Program Definition span (x : ptr) : bool = {
  if x == null then return false;
  else
    b ← CAS(x->m, 0, 1);
    if b then
      (rₗ,rᵣ) ← (span(x->l) || span(x->r));
      if ¬rₗ then x->l := null;
      if ¬rᵣ then x->r := null;
      return true;
    else return false;
}
```

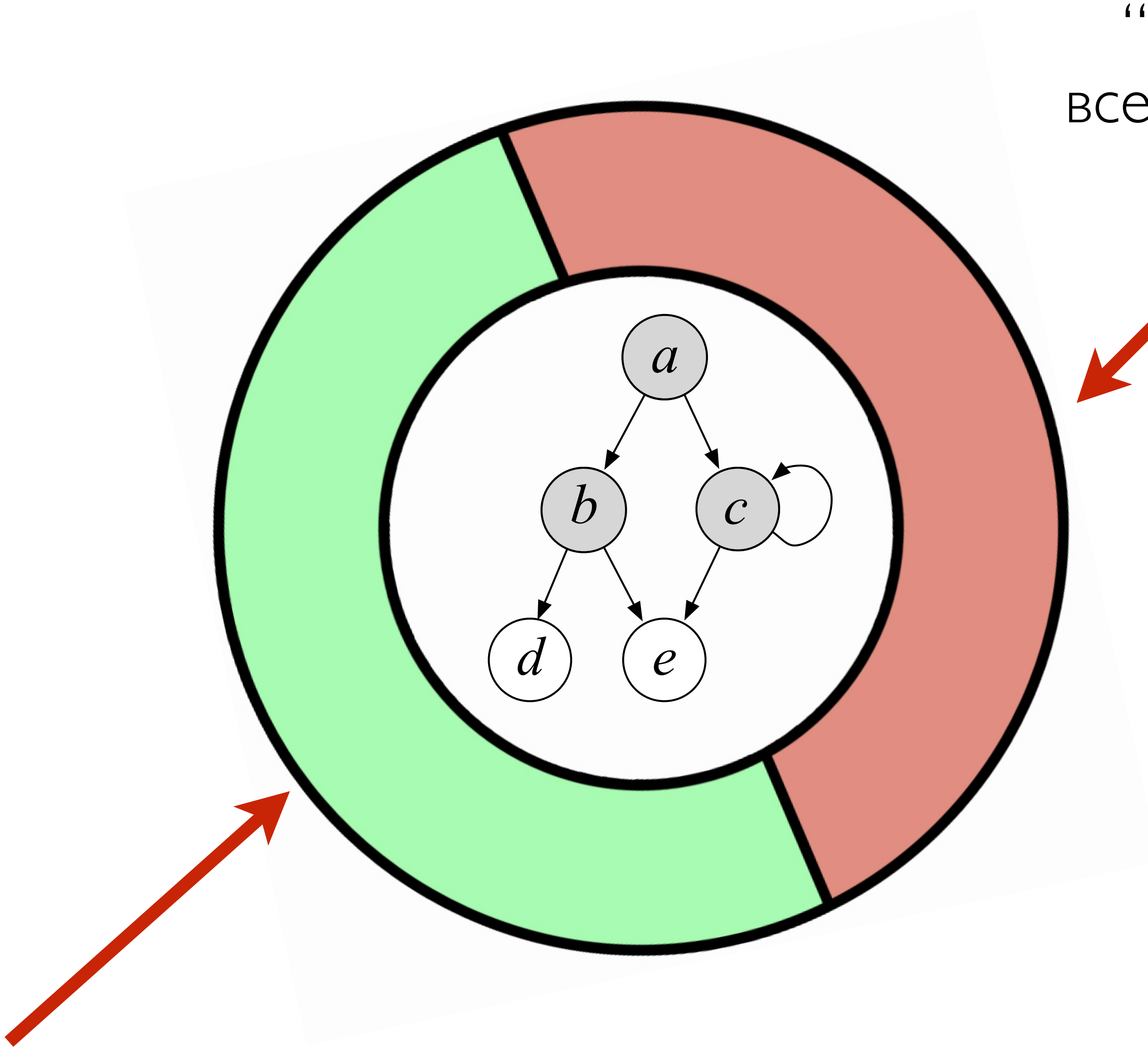
- Все достижимые вершины графа в итоге помечены
- Никакие “сторонние” процессы не изменяют граф
- Вызов из корневой вершины сделан одним “изначальным” потоком.

Модель разделяемой памяти



Модель разделяемой памяти

“Фиктивная” память
всех остальных потоков



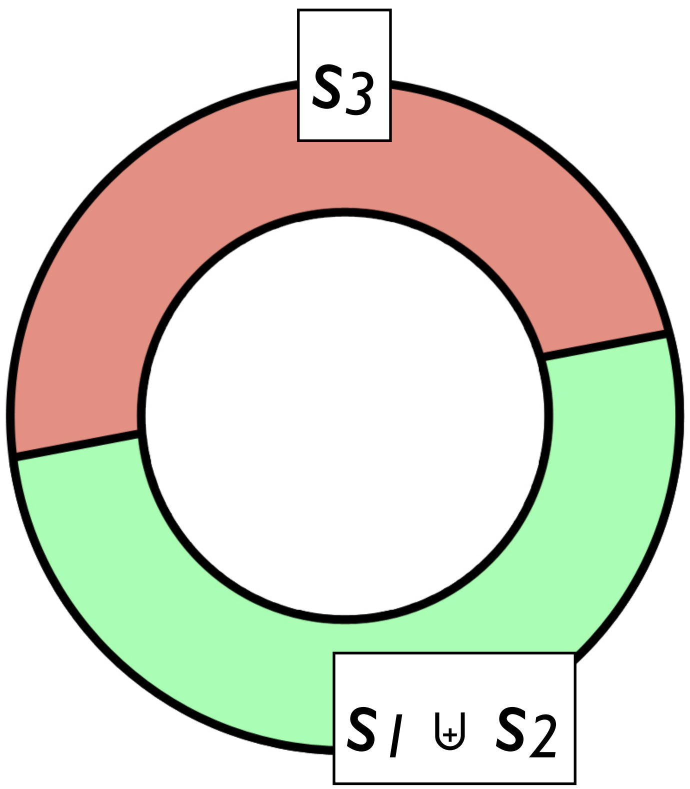
“Фиктивная” память
конкретного потока

Разделение фиктивной памяти

```
span(x)

span(x->l) || span(x->r)
```

Разделение фиктивной памяти

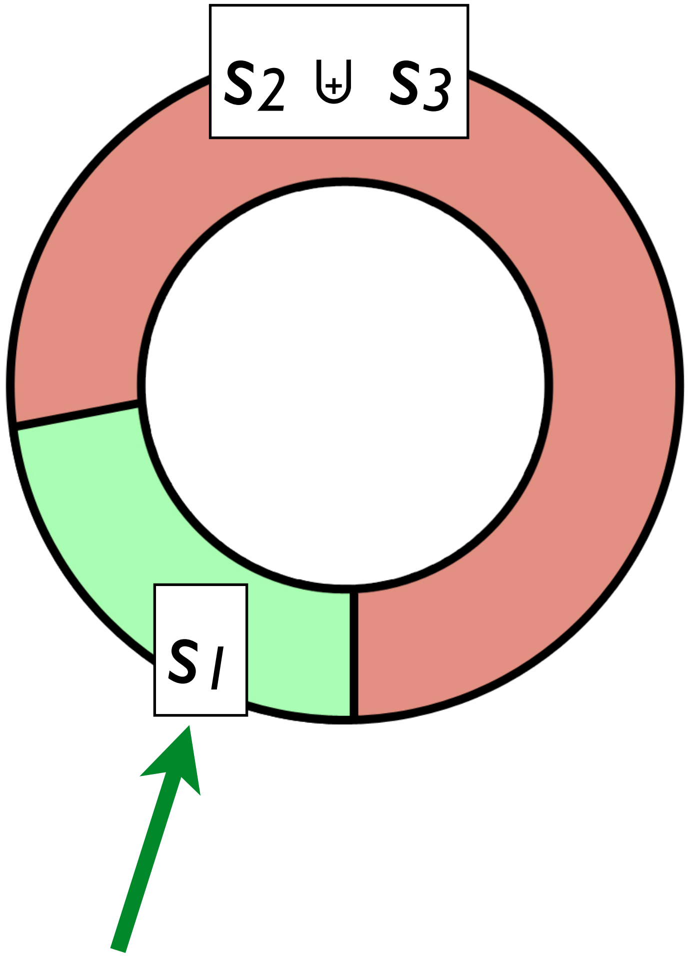


`span(x)`

$\{ s_1 \uplus s_2 \}$

`span(x->l)` $\Vert$ `span(x->r)`

Разделение фиктивной памяти



Вершины, "отданные" `span(x->l)`

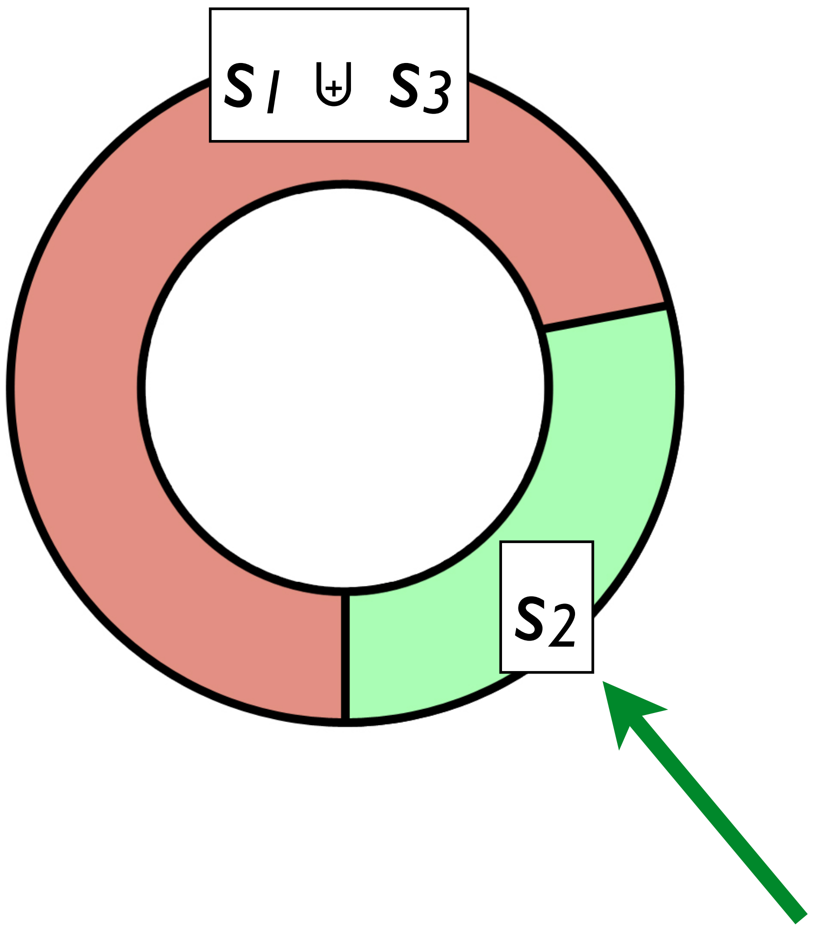
`span(x)`

$\{\ s_l \uplus s_2\ \}$

$\{\ s_l\ \}$

`span(x->l)` || `span(x->r)`

Разделение фиктивной памяти



`span(x)`

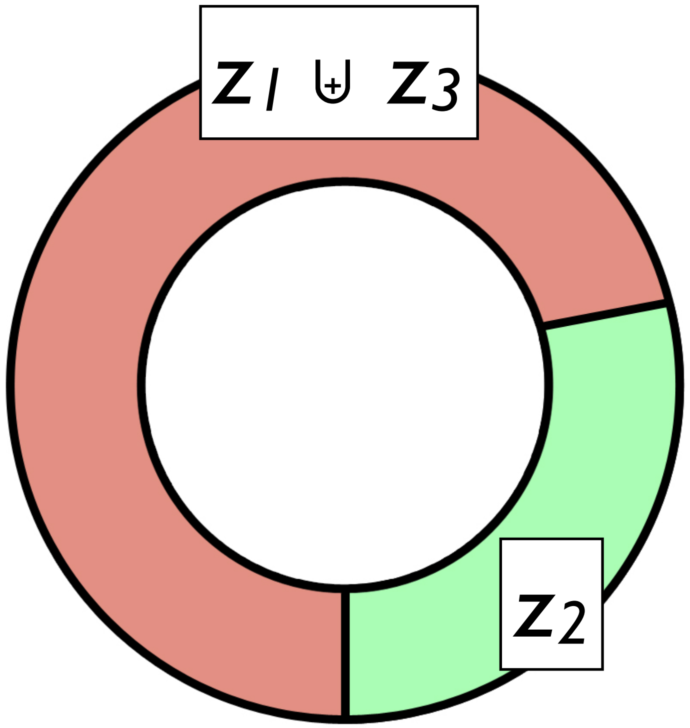
$\{\ s_1 \uplus s_2\ \}$

$\{\ s_1\ \}$ $\qquad\qquad$ $\{\ s_2\ \}$

`span(x->l)` || `span(x->r)`

Вершины, "отданные" `span(x->r)`

Разделение фиктивной памяти



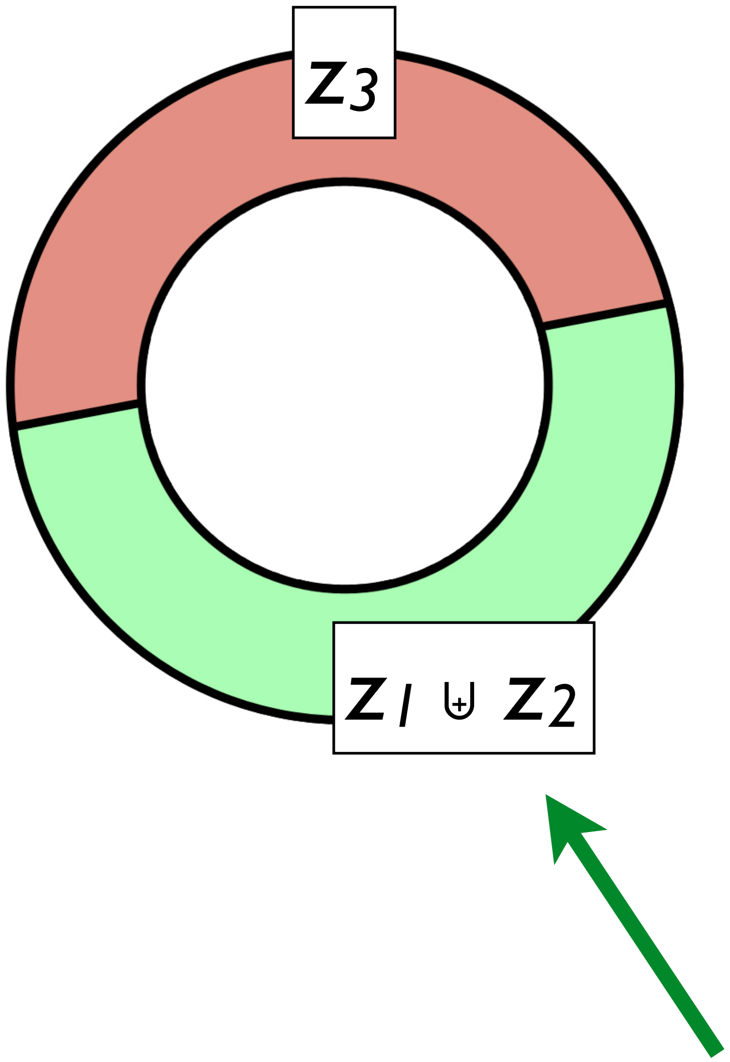
`span(x)`

$\{ s_1 \uplus s_2 \}$

$$\frac{\{ s_1 \}\ \texttt{span(x->l)}\ \{ z_1 \} \qquad \{ s_2 \}\ \texttt{span(x->r)}\ \{ z_2 \}}{} $$

$\{ s_1 \}$		$\{ s_2 \}$
`span(x->l)`	$\Vert$	`span(x->r)`
$\{ z_1 \}$		$\{ z_2 \}$

Разделение фиктивной памяти



Вершины, помеченные `span(x)`

`span(x)`

$\{\, s_1 \uplus s_2 \,\}$

$\{\, s_1 \,\}$ $\qquad$ $\{\, s_2 \,\}$

`span(x->l)` $\parallel$ `span(x->r)`

$\{\, z_1 \,\}$ $\qquad$ $\{\, z_2 \,\}$

$\{\, z_1 \uplus z_2 \,\}$

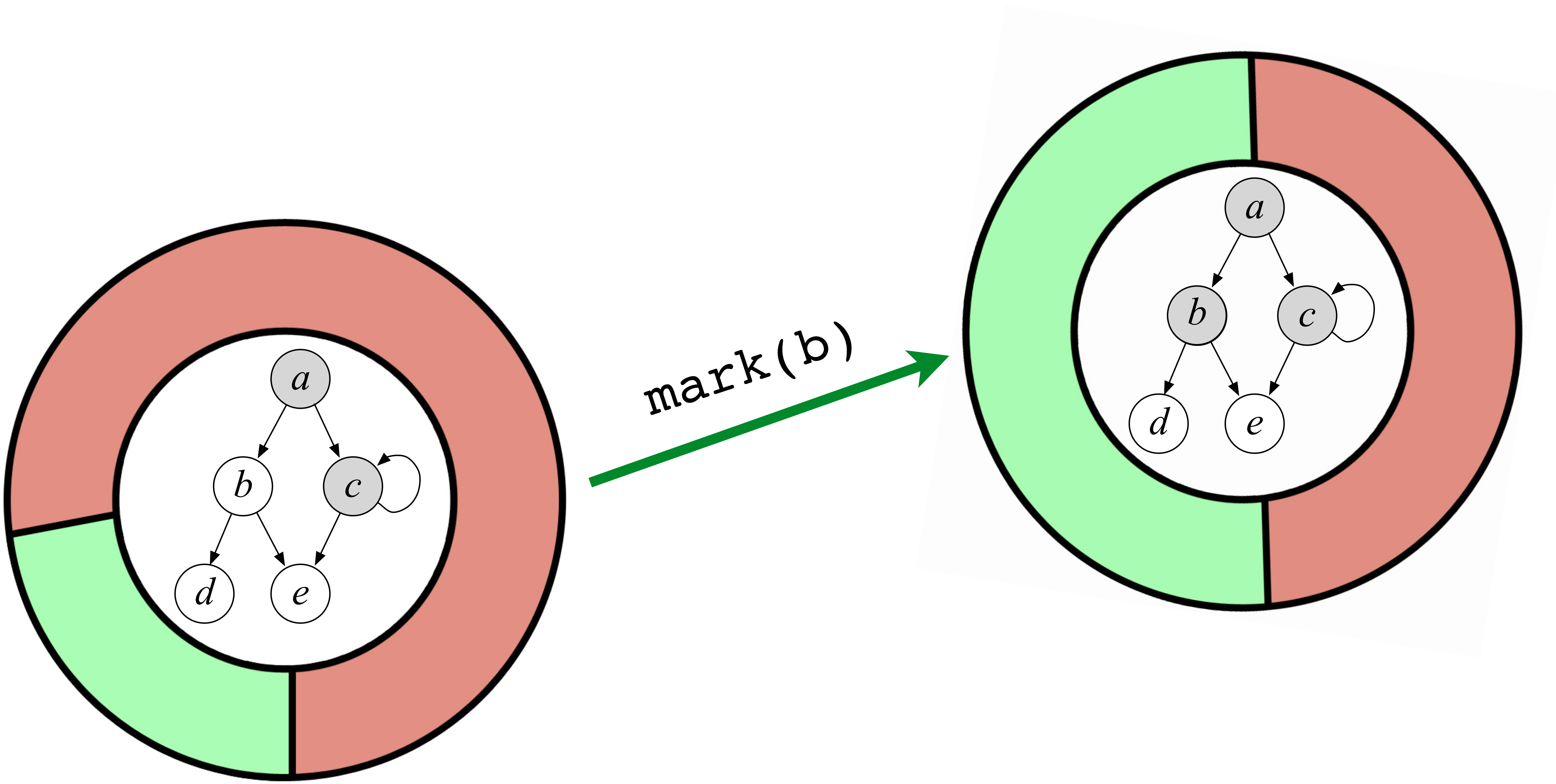
`span(x)`

Верификация span

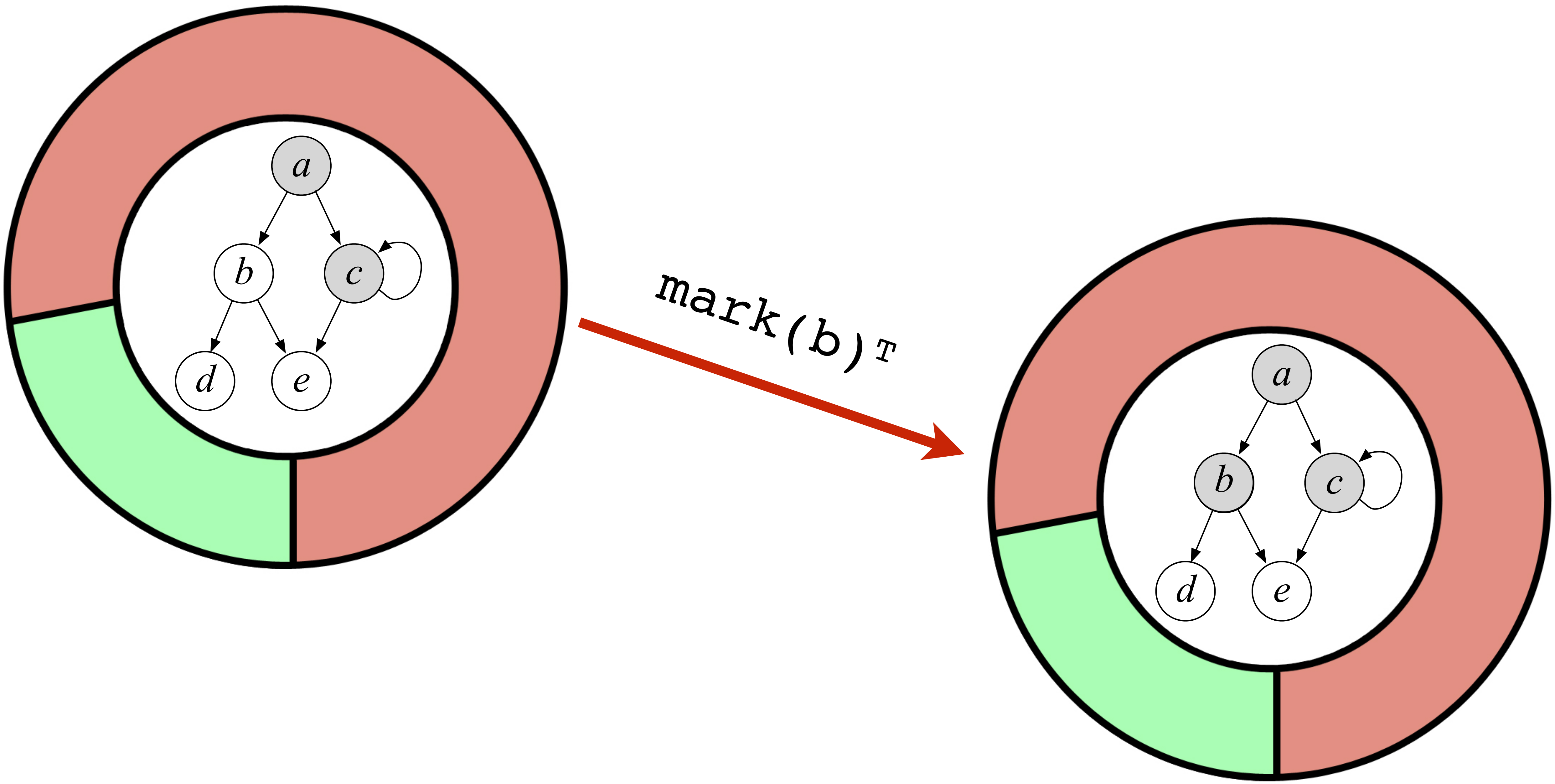
```
Program Definition span (x : ptr) : bool = {
  if x == null then return false;
  else
    b ← CAS(x->m, 0, 1);
    if b then
      (r_l,r_r) ← (span(x->l) || span(x->r));
      if ¬r_l then x->l := null;
      if ¬r_r then x->r := null;
      return true;
    else return false;
}
```

- Все достижимые вершины графа в итоге помечены
- Никакие “сторонние” процессы не изменяют граф
- Вызов из корневой вершины сделан одним “изначальным” потоком.

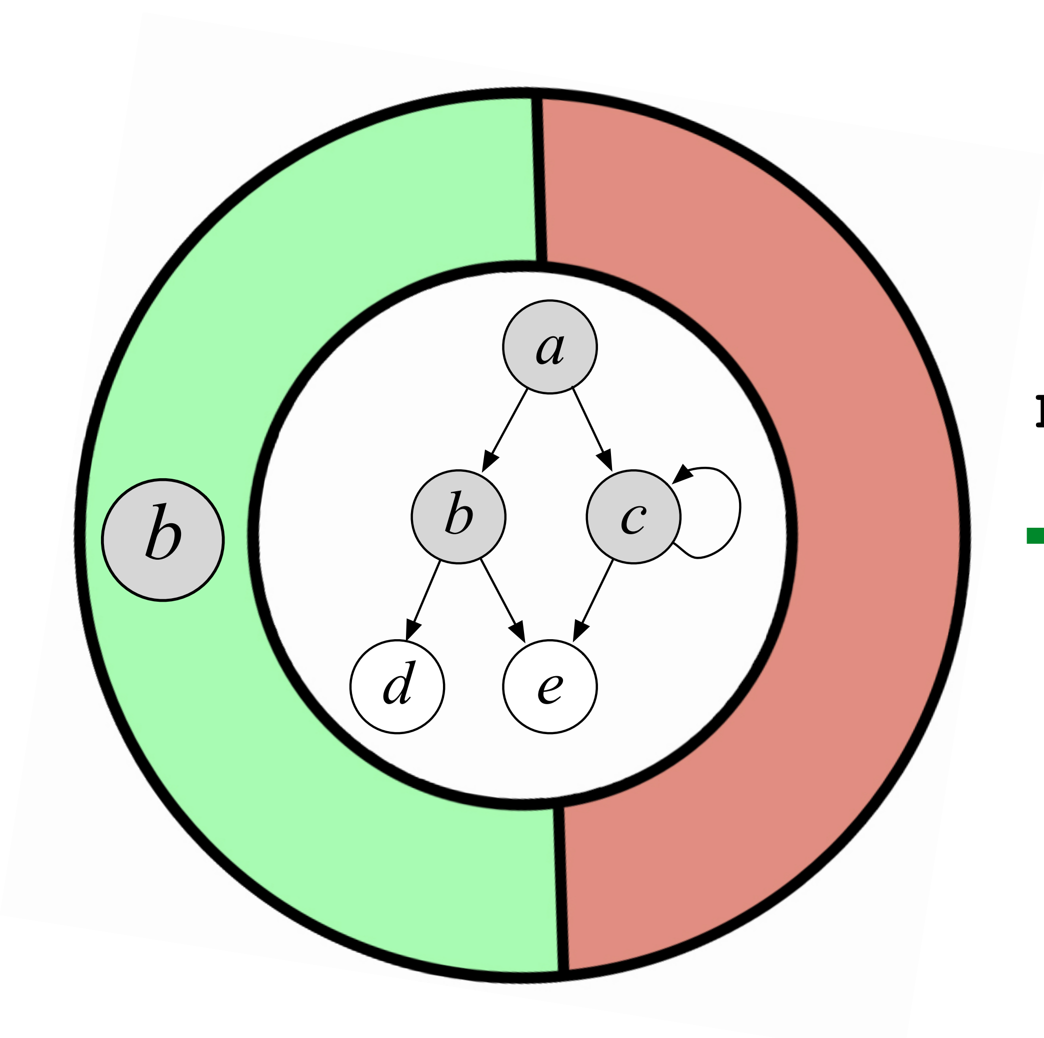
Соглашение 1: пометка вершины



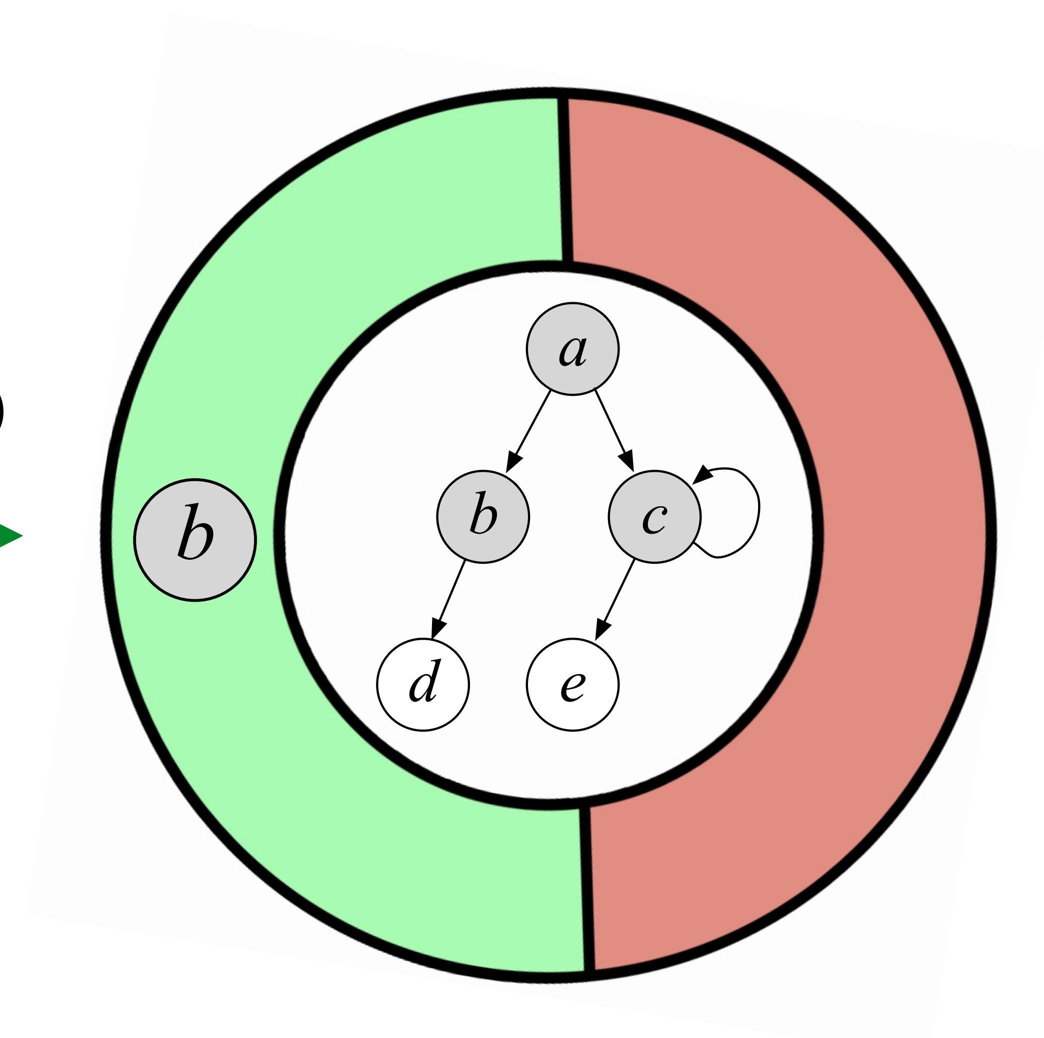
Соглашение 1: пометка вершины



Соглашение 2: удаление ребра



`nullify(b->r)`



```
Program Definition span (x : ptr) : bool = {
  if x == null then return false;
  else
    b ← CAS(x->m, 0, 1);
    if b then
      (r_l,r_r) ← (span(x->l) || span(x->r));
      if ¬r_l then x->l := null;
      if ¬r_r then x->r := null;
      return true;
    else return false;
}
```

```
Program Definition span : span_tp :=
 ffix (fun (loop : span_tp) (x : ptr) =>
   Do (if x == null then ret false else
       b <-- trymark x;
       if b then
         xl <-- read_child x Left;
         xr <-- read_child x Right;
         rs <-- par (loop xl) (loop xr);
         (if ~~rs.1 then nullify x Left else ret tt);;
         (if ~~rs.2 then nullify x Right else ret tt);;
         ret true
       else ret false)).
```

Зависимые типы
для многопоточности

λ calculus
A. Church (1930s)

Turing Machine
A. Turing (1936)

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$$\{\ P\ \}\ c\ \{\ Q\ \}\ @R$$

$$\{\ P\ \}\ \texttt{span(x)}\ \{\ Q\ \}@R_{SpanTree}$$

$$\texttt{span(x) : span\_tp}\ (x, R_{SpanTree}, P\ , Q)$$

Тип/Спецификация

```
Program Definition span : span_tp :=
 ffix (fun (loop : span_tp) (x : ptr) =>
   Do (if x == null then ret false else
      b <-- trymark x;
      if b then
        xl <-- read_child x Left;
        xr <-- read_child x Right;
        rs <-- par (loop xl) (loop xr);
        (if ~~rs.1 then nullify x Left else ret tt);;
        (if ~~rs.2 then nullify x Right else ret tt);;
        ret true
      else ret false)).
```

Proof. (около 200 LOC) **Qed.**

стартовая вершина

```
Definition span_tp (x : ptr) :=
  {i (g1 : graph (joint i))}, STsep [SpanTree]

    (fun s1 => i = s1 ∧ (x == null ∨ x ∈ dom (joint s1)),

     fun (r : bool) s2 => exists g2 : graph (joint s2),
       subgraph g1 g2 ∧
       if r then x != null ∧
                exists (t : set ptr),
                  self s2 = self i ⊎ t ∧
                  tree g2 x t ∧
                  maximal g2 t ∧
                  front g1 t (self s2 ⊎ other s2)
       else (x == null ∨ mark g2 x) ∧
            self s2 = self i).
```

протокол взаимодействия

```
Definition span_tp (x : ptr) :=
  {i (g1 : graph (joint i))}, STsep [SpanTree]

    (fun s1 => i = s1 ∧ (x == null ∨ x ∈ dom (joint s1)),

     fun (r : bool) s2 => exists g2 : graph (joint s2),
       subgraph g1 g2 ∧
       if r then x != null ∧
                 exists (t : set ptr),
                   self s2 = self i ⊎ t ∧
                   tree g2 x t ∧
                   maximal g2 t ∧
                   front g1 t (self s2 ⊎ other s2)
       else (x == null ∨ mark g2 x) ∧
            self s2 = self i).
```

предусловие

```
Definition span_tp (x : ptr) :=
  {i (g1 : graph (joint i))}, STsep [SpanTree]

    (fun s1 => i = s1 ∧ (x == null ∨ x ∈ dom (joint s1)),

     fun (r : bool) s2 => exists g2 : graph (joint s2),
       subgraph g1 g2 ∧
       if r then x != null ∧
                exists (t : set ptr),
                  self s2 = self i ⊎ t ∧
                  tree g2 x t ∧
                  maximal g2 t ∧
                  front g1 t (self s2 ⊎ other s2)
       else (x == null ∨ mark g2 x) ∧
            self s2 = self i).
```

```
Definition span_tp (x : ptr) :=
  {i (g1 : graph (joint i))}, STsep [SpanTree]

    (fun s1 => i = s1 ∧ (x == null ∨ x ∈ dom (joint s1)),

     fun (r : bool) s2 => exists g2 : graph (joint s2),
       subgraph g1 g2 ∧
       if r then x != null ∧
                 exists (t : set ptr),
                   self s2 = self i ⊎ t ∧
                   tree g2 x t ∧
                   maximal g2 t ∧
                   front g1 t (self s2 ⊎ other s2)
       else (x == null ∨ mark g2 x) ∧
            self s2 = self i).
```

постусловие

```
Definition span_tp (x : ptr) :=
  {i (g1 : graph (joint i))}, STsep [SpanTree]

    (fun s1 => i = s1 ∧ (x == null ∨ x ∈ dom (joint s1)),

     fun (r : bool) s2 => exists g2 : graph (joint s2),
       subgraph g1 g2 ∧
       if r then x != null ∧
                 exists (t : set ptr),
                   self s2 = self i ⊎ t ∧
                   tree g2 x t ∧
                   maximal g2 t ∧
                   front g1 t (self s2 ⊎ other s2)
       else (x == null ∨ mark g2 x) ∧
            self s2 = self i).
```

```
Definition span_tp (x : ptr) :=
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    (fun s1 => i = s1 ∧ (x == null ∨ x ∈ dom (joint s1)),

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       subgraph g1 g2 ∧
       if r then x != null ∧
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       else (x == null ∨ mark g2 x) ∧
            self s2 = self i).
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Definition span_tp (x : ptr) :=
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                   self s2 = self i ⊎ t ∧
                   tree g2 x t ∧
                   maximal g2 t ∧
                   front g1 t (self s2 ⊎ other s2)
       else (x == null ∨ mark g2 x) ∧
            self s2 = self i).
```

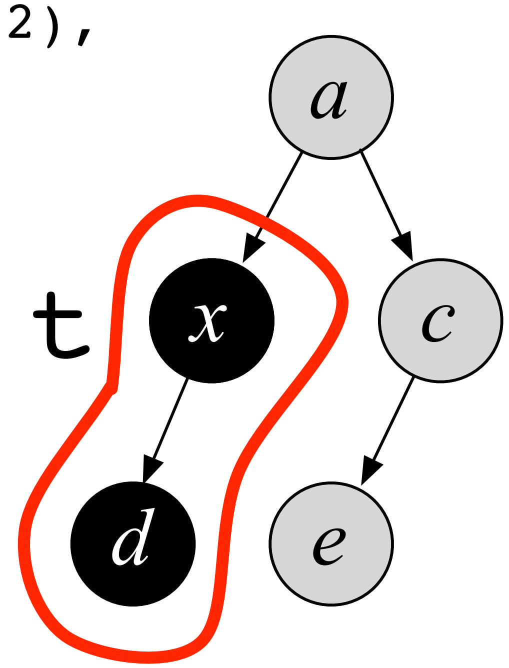

```
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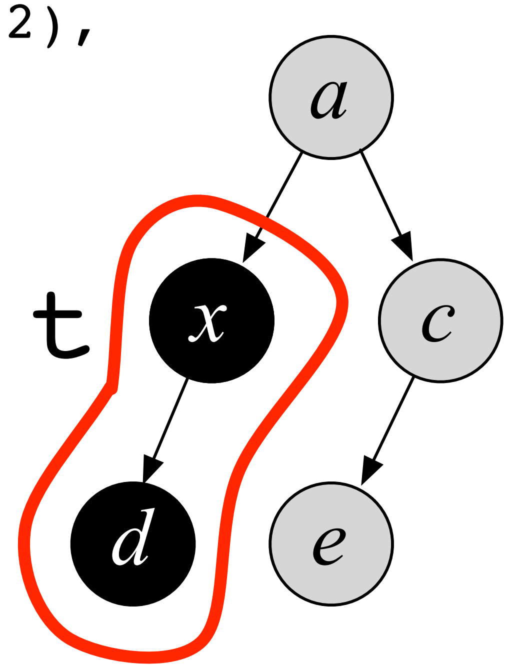

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       if r then x != null ∧
                 exists (t : set ptr),
                   self s2 = self i ⊎ t ∧
                   tree g2 x t ∧
                   maximal g2 t ∧
                   front g1 t (self s2 ⊎ other s2)
       else (x == null ∨ mark g2 x) ∧
            self s2 = self i).
```

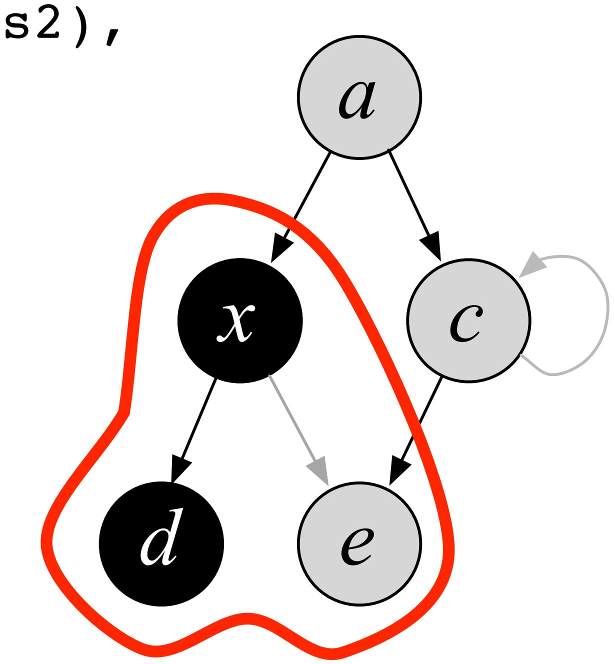

```
Program Definition span (x : ptr) : bool = {
  if x == null then return false;
  else
    b ← CAS(x->m, 0, 1);
    if b then
      (rₗ,rᵣ) ← (span(x->l) || span(x->r));
      if ¬rₗ then x->l := null;
      if ¬rᵣ then x->r := null;
      return true;
    else return false;
}
```

- Все достижимые вершины графа в итоге помечены
- Никакие "сторонние" процессы не изменяют граф
- Вызов из корневой вершины сделан одним "изначальным" потоком.

```
Definition span_tp (x : ptr) :=
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     fun (r : bool) s2 => exists g2 : graph (joint s2),
       subgraph g1 g2 ∧
       if r then x != null ∧
                 exists (t : set ptr),
                   self s2 = self i ⊎ t ∧
                   tree g2 x t ∧
                   maximal g2 t ∧
                   front g1 t (self s2 ⊎ other s2)
       else (x == null ∨ mark g2 x) ∧
            self s2 = self i).
```
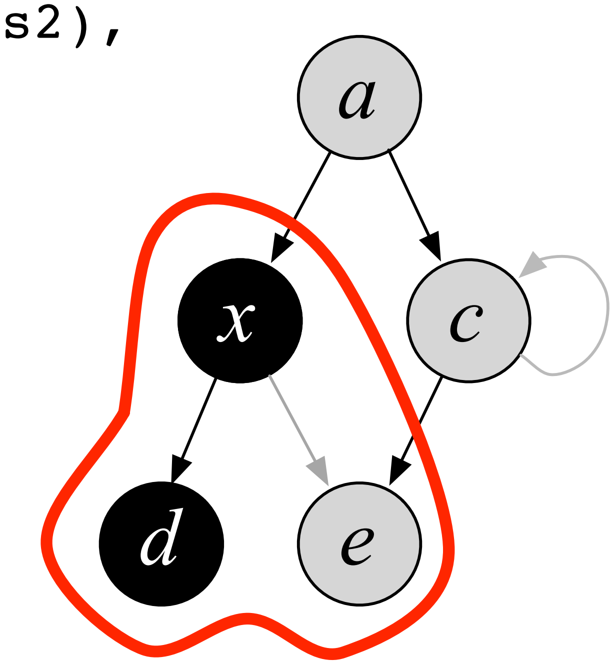
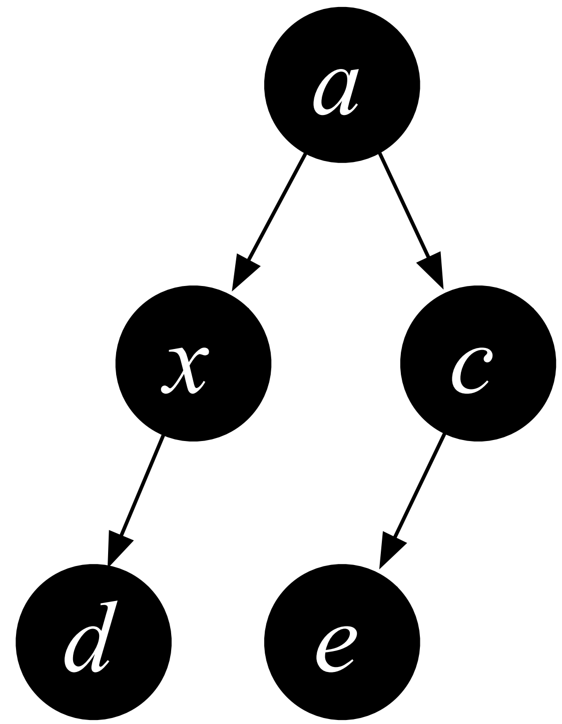



Предполагает эффект параллельных потоков

Изначально только 1 поток



следствия постусловия и связности графа

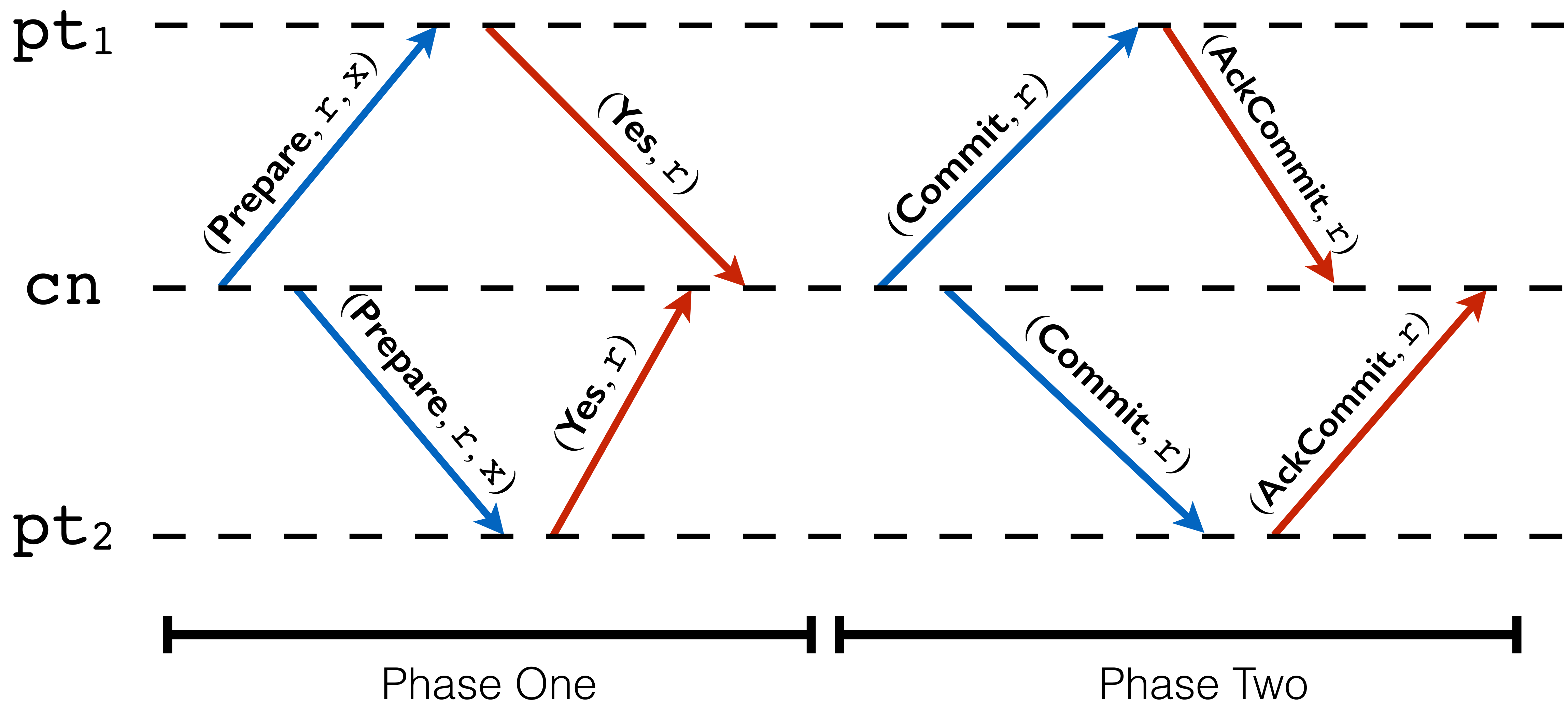
```
tree g2 a t            ∧ maximal g2 t ∧
front g1 t (self s2) ∧ t = self s2  ∧
is_root a g1           ∧ subgraph g1 g2
⇒ spanning t g1
```

Checkpoint 3: Зависимые типы для многопоточных программ

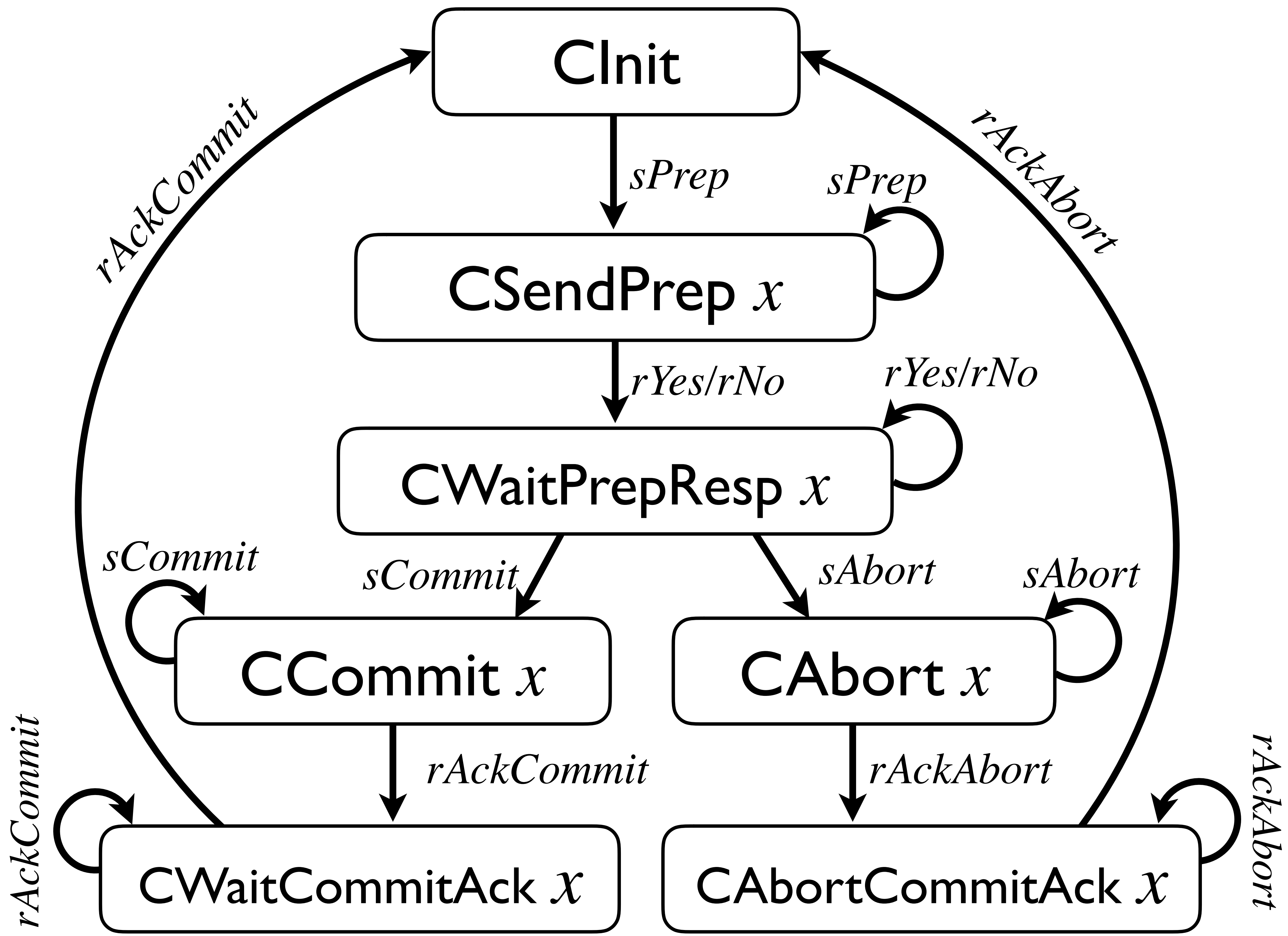
- Типы описывают *состояние разделенной памяти*;
- Типы также описывают *протокол* взаимодействия;
- Проверка типов (верификация) гарантирует, что *все потоки соблюдают протокол* и корректно делят “фиктивную” память.

Зависимые типы
для распределенных вычислений

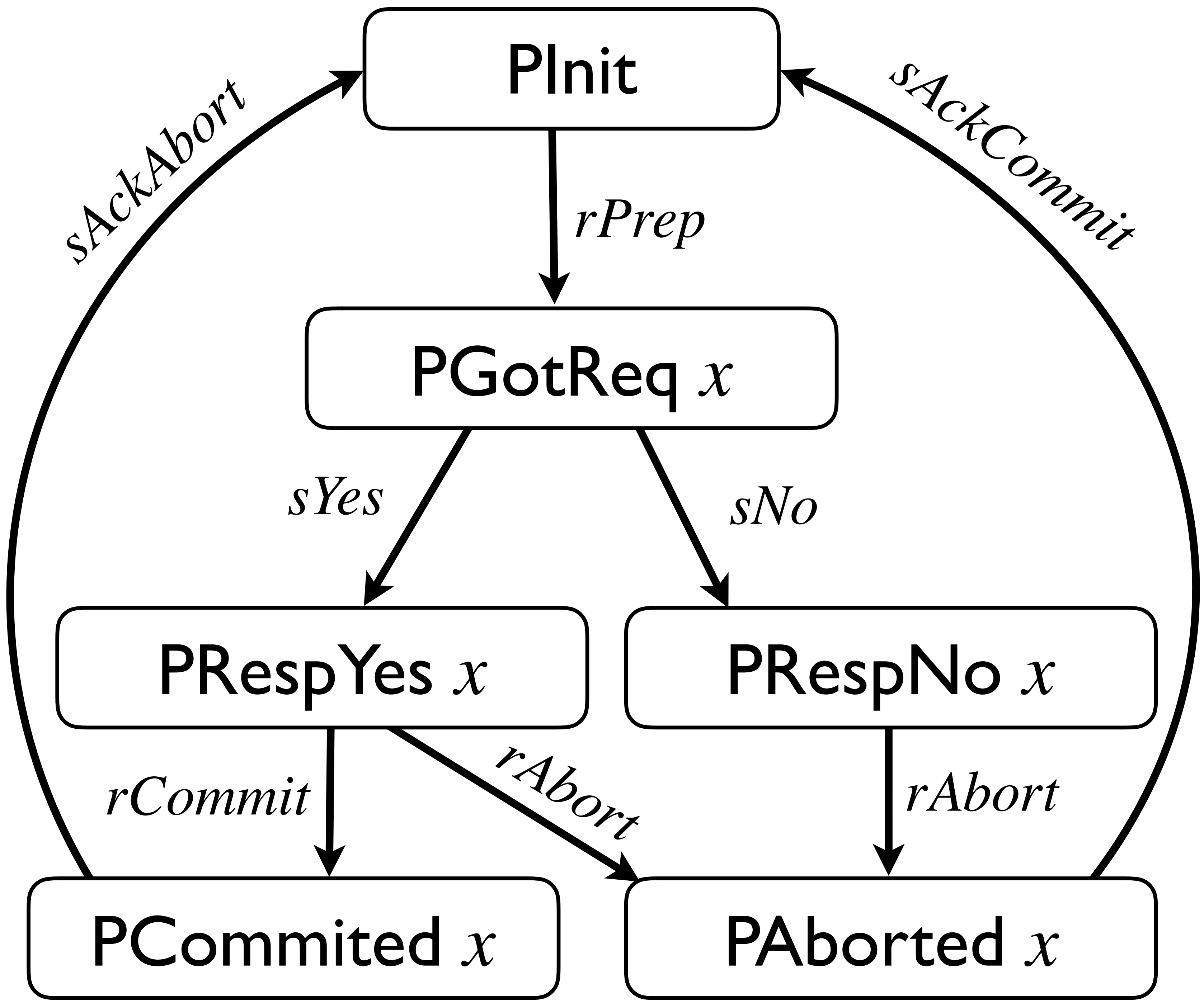
Two-Phase Commit Protocol



Состояния “Координатора”



Состояния “Участника"



Изменения в состоянии участника

Отправка сообщения

```
Definition p_send_step (r: round) (ps: PState) (log: Log)
           (commit: bool): round * PState * Log :=
  | PGotReq x ⇒ if commit then (r, PRespYes x, log)
                          else (r, PRespNo x, log)
  (* ... more cases depending on ps ... *)
  end.
```

Получение сообщения

```
Definition p_recv_step (r: round) (ps: PState)
           (log: Log) (tag: nat) (mbody: seq nat) :=
  | PRespYes x ⇒ if tag == Commit
      then (r + 1, PCommitted x, log ++ [(true, x)])
      else (r + 1, PAborted x, log ++ [(false, x)])
  (* ... more cases depending on ps, tag, mbody ... *)
  end.
```

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Distributed Hoare Types
I. Sergey (2017)

Реализация участника

```
Program Definition participant_round (commit: bool) :
    {(r: round) (log: Log)}, DHT [pt, W]

      (fun s ⇒  loc pt s = st ↦ (r, PInit) ⊎

                           lg ↦ log,

       fun _ s’ ⇒ ∃ (b: bool) (x: data),

                loc pt s’ = st ↦ (r+1, PInit) ⊎
                            lg ↦ (log ++ [(b, x)]))
 :=
    Do (r ← read_round;
        receive_prepare_req r;
        respond_to_req r commit;
        e ← receive_commit_or_abort r;
        send_ack e).
```

Инвариант системы 2PC

```
Lemma pt_log_agreement d r log pt :
  coh d → pt ∈ pts → Inv d →
  loc pt d = st ↦ (r, PInit) ⊎ lg ↦ log →
  ∀ pt' ps' log', pt' ∈ pts →
    loc pt' d = st ↦ (r, ps') ⊎ lg ↦ log' → log' = log.
```

Между раундами "истории" участников *совпадают*.

Участник как библиотека

```
Definition run_participant (choices: seq bool): DHT [pt, _]
      (fun i ⇒ i = init_state,
       fun _ m ⇒ ∃ r (results : seq bool) (stream: seq data),
           let log := zip results stream in
           loc pt m = st ↦ (r, PInit) ⊎
                      lg ↦ log ∧
            ∀ pt’ ps’ lg’, pt’ ∈ pts →
               loc pt’ m = st ↦ (r, ps’) ⊎
                           lg ↦ log’ → log = log’)
:=
   participant choices.
```

Checkpoint 4: Зависимые типы для распределённых приложений

- Типы описывают *протокол взаимодействия*, представленный системой переходов;
- Типы могут описывать состояние *многих* реплик;
- Проверка типов (верификация) гарантирует, что все участники соблюдают протокол, равно как и *сохранение инвариантов* системы.

В заключение

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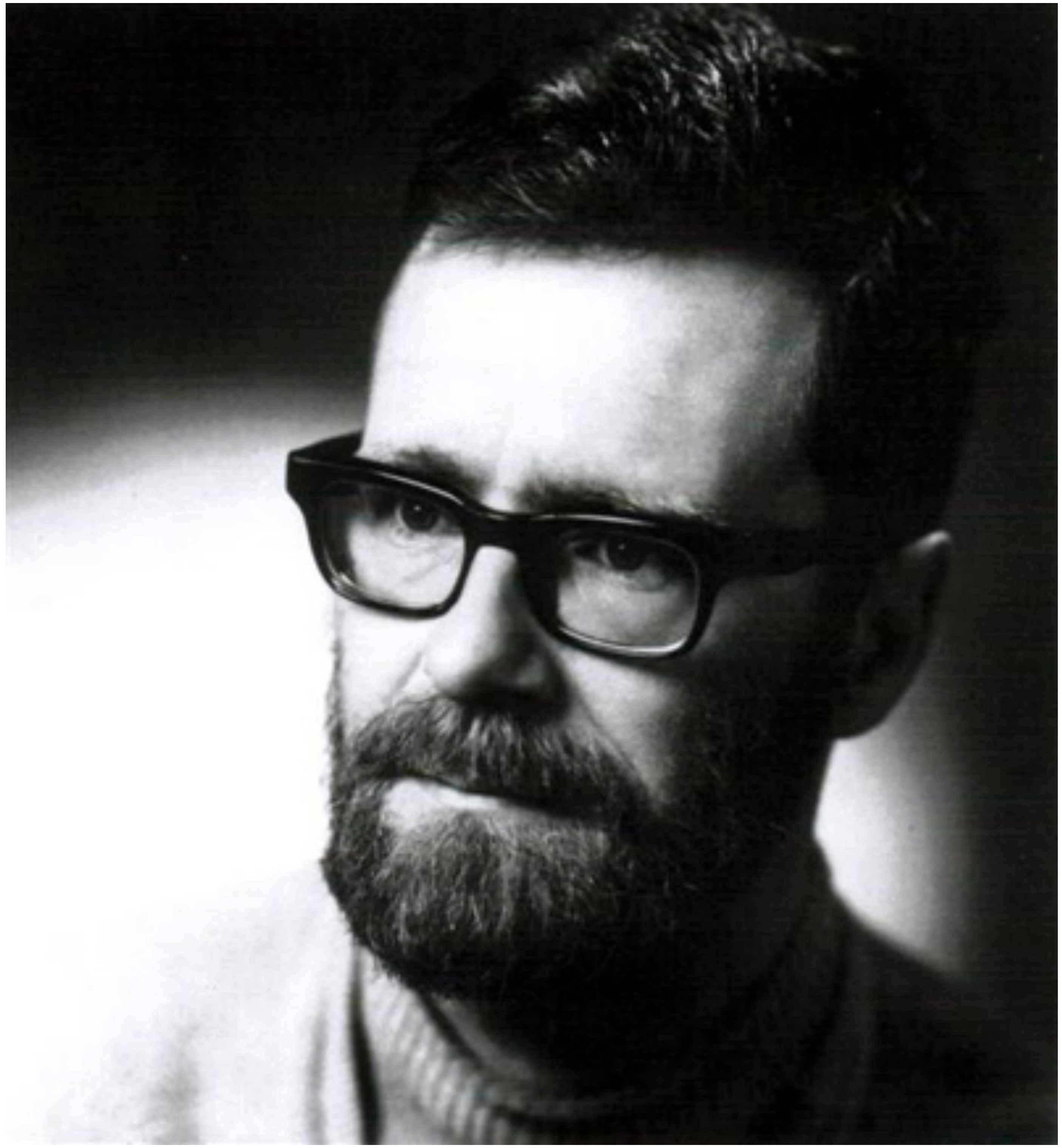
A. Nanevski, I. Sergey (2013-15)

Distributed Hoare Types
I. Sergey (2017)

- Типизированные программы *не содержат ошибок*;
- Зависимые типы — отсутствие *нетривиальных* ошибок;
- Логики программ (**Program Logics**) — спецификация и верификация *императивного* кода;
- **Separation Logic + Dependent Types = Hoare Types** — типы для программ с *управлением памятью*;
- *Многопоточные* и *распределенные* вычисления: Hoare Types + расширенная модель состояния.



Edsger W. Dijkstra

On the cruelty of really teaching computing science

Edsger W. Dijkstra

We stress that the programmer's task is ***not just to write*** down a program, but that his main task is to ***give a formal proof*** that the program he proposes meets the ***formal functional specification***. […]

The rules of proof manipulation are ***so few and simple*** that very soon thereafter he makes the ***exciting discovery*** that he is beginning to master the use of ***a tool that, in all its simplicity, gives us a power*** that far surpasses his wildest dreams.

Спасибо за внимание!