From Type Checking by Recursive Descent to Type Checking by Abstract Machine

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This work was carried out while the first author was visiting the BRICS PhD School of Aarhus University in September 2010
What is a program semantics?
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It is the meaning of grammatically correct programs.
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Example of a meaning:
What is a program semantics?

It is the meaning of grammatically correct programs

Example of a meaning: types
Outline

A parade of semantics

Semantics of type checking
  Derivation rules for type checking
  An abstract machine
  Reduction semantics

Semantics equivalence problem

Functional transformation
  Toolbox
  Inter-derivation

Summary and conclusion
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A parade of semantics

Different semantics are aimed to answer different questions about programs:

- **Denotational semantics**: what does a program mean as a mathematical object. 
  - C. Strachey, D. Scott
- **Operational semantics**: how to compute a program on some abstract machine, what is its result.
  - G. Plotkin
- **Axiomatic semantics**: what are properties of the effect of executing a program.
  - R.W. Floyd, C.A.R. Hoare
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*Denotational semantics* gives an intuition about “what a program is”, but doesn’t say how to execute it.
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Operational semantics defines how to execute a program.
A diversity of operational semantics

- **Big-step (natural) semantics:**
  - program evaluation defined inductively on its syntax
  - computes a *fold* over a program’s AST

- **Big-step abstract machine**
  - Execution traces instead of trees

- **Small-step operational semantics**
  - Each step: decompose-contract-recompose

- **Reduction semantics**
  - Contexts and contractions

- **Small-step abstract machine**
  - Examples: CC, SCC, CK, CEK-machines, Krivine’s machine, Landin’s SECD etc.
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A diversity of semantic artifacts

Semantics are described via some *meta-languages*
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Any expressive *programming language* may play a role of a *meta-language*
A diversity of semantic artifacts

Semantics are described via some _meta-languages_

Any expressive _programming language_
may play a role of a _meta-language_

therefore

_Semantic formalisms_ can be directly represented as _programs_
(or _semantic artifacts_)
Semantics equivalence problem

All these semantics were developed independently of each other.
Semantics equivalence problem

All these semantics were developed independently of each other

Their equivalence should be proved
Semantics equivalence problem

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Can we connect them some other way?
Calculational inter-derivation

Program *semantics* can be connected via the inter-derivation of the corresponding *semantic artifacts*.\(^1\)

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\(^1\)O. Danvy, ICFP '08
This connection has never been done for *type checking*
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Type checking: description I

*Type checking* is a semantics of a “typing language” on top of the host language’s syntax.
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*Type checking* is a semantics of a “typing language” on top of the host language’s syntax.

Its natural semantics is a familiar one, in the form of *derivation rules*.

\[
\begin{align*}
\text{[t-lam]} & \quad \begin{array}{c}
\Gamma[x : \tau_1] \vdash e : \tau_2 \\
\hline
\Gamma \vdash \lambda x : \tau_1. e : \tau_1 \to \tau_2
\end{array} \\
\text{[t-var]} & \quad \begin{array}{c}
(x : \tau \in \Gamma) \\
\hline
\Gamma \vdash x : \tau
\end{array} \\
\text{[t-app]} & \quad \begin{array}{c}
\Gamma \vdash e_1 : \tau_1 \to \tau_2 \\
\Gamma \vdash e_2 : \tau_1 \\
\hline
\Gamma \vdash e_1 e_2 : \tau_2
\end{array} \\
\text{[t-num]} & \quad \begin{array}{c}
\Gamma \vdash \text{number} : \text{num}
\end{array}
\end{align*}
\]

Type system for the simply typed lambda calculus with numbers
Another semantics of typing language: a *small-step abstract machine with control and result stacks* (SEC-machine)

\[
\begin{align*}
\langle S, E, \text{num}:C \rangle & \Rightarrow_t \langle \text{num}:S, E, C \rangle \\
\langle S, E[x\mapsto \tau], x:C \rangle & \Rightarrow_t \langle \tau: S, E[x\mapsto \tau], C \rangle \\
\langle S, E, (\lambda x: \tau. e):C \rangle & \Rightarrow_t \langle \text{nil}, E\sqcup \{x\mapsto \tau\}, e: \text{Lam}(\tau, S):C \rangle \\
\langle S, E, (e_1 e_2):C \rangle & \Rightarrow_t \langle S, E, e_1: \text{Fun}(e_2):C \rangle \\
\langle \tau_2: S, E, \text{Lam}(\tau_1, S') : C \rangle & \Rightarrow_t \langle (\tau_1 \rightarrow \tau_2): S', E, C \rangle \\
\langle (\tau_1 \rightarrow \tau_2): S, E, \text{Fun}(e_2):C \rangle & \Rightarrow_t \langle (\tau_1 \rightarrow \tau_2): S, E, e_2: \text{Arg}(\tau_1, \tau_2):C \rangle \\
\langle \tau_1: x: S, E, \text{Arg}(\tau_1, \tau_2):C \rangle & \Rightarrow_t \langle \tau_2: S, E, C \rangle 
\end{align*}
\]

\(^2\)C. Hankin and D. Le Météayer, POPL ’94
Another semantics of typing language: a *small-step abstract machine with control and result stacks* (SEC-machine)\(^2\)

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\langle \tau_1:x:S, E, \text{Arg}(\tau_1, \tau_2):C \rangle & \Rightarrow_t \langle \tau_2:S, E, C \rangle
\end{align*}
\]
And yet another one: *reduction semantics*\(^3\)

\[
e ::= \ n \mid x \mid \lambda x:\tau.e \mid e \ e \mid \tau \rightarrow e \mid \text{num}
\]

\[
T ::= \ T \ e \mid \tau \ T \mid \tau \rightarrow T \mid [ ]
\]

\[
\tau ::= \ \text{num} \mid \tau \rightarrow \tau
\]

\[
n ::= \ \text{number}
\]

Hybrid language and type-checking contexts

\[
T[n] \mapsto_t T[\text{num}] \quad [\text{tc-const}]
\]

\[
T[\lambda x:\tau.e] \mapsto_t T[\tau \rightarrow \{\tau/x\} \ e] \quad [\text{tc-lam}]
\]

\[
T[(\tau_1 \rightarrow \tau_2) \ \tau_1] \mapsto_t T[\tau_2] \quad [\text{tc-\tau\beta}]
\]

Type-checking reduction rules

\(^3\)G. Kuan, D. MacQueen and R. B. Findler, ESOP '07
Why should we care about different semantics?

Given formalism for a type systems defines a corresponding semantic artifact.
Why should we care about different semantics?

Given formalism for a type systems defines a corresponding semantic artifact

- Type derivation rules $\sim$ recursive descent
- Machine-like semantics $\sim$ driver-loop abstract machine (CEK, SECD etc.)
- Reduction semantics $\sim$ \texttt{decompose-contract-recompose} loop
Why should we care about different semantics?

Given formalism for a type systems defines a corresponding *semantic artifact*

- Type derivation rules \(\sim\) recursive descent
- Machine-like semantics \(\sim\) driver-loop abstract machine (CEK, SECD etc.)
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Benefits of non-standard semantics:
- type debugging
- optimized computation
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Semantics equivalence again

Do all these semantics describe the same type checking procedure?
Semantics equivalence again

Do all these semantics describe the same type checking procedure?

**Theorem** [Hankin and Le Métayer] (Soundness and Completeness for $\Rightarrow_t$)
\[ \Gamma \vdash e : \tau \iff \langle S, \Gamma, e : C \rangle \Rightarrow_t \langle \tau : S, \Gamma, C \rangle. \]

**Theorem** [Kuan et al.] (Soundness and Completeness for $\mapsto^*_t$)
For any $e$ and $\tau$, $\emptyset \vdash e : \tau$ iff $e \mapsto^*_t \tau$
Our concern

Can we \textit{inter-derive} these semantics \textit{a priori} rather than \textit{prove} their equivalence \textit{a posteriori}?
Our contribution

Yes, we can.
Our contribution

Yes, we can.

via functional inter-derivation

Transformations instead of proofs
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Two approaches

A mathematician’s approach: to prove the equivalence between semantics by induction (or bisimulation by coinduction)

A programmer’s approach:
- take one particular implementation
- apply a series of transformations to a program
- be sure that transformations are correct
Two approaches

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A programmer’s approach:
- take one particular implementation
- apply a series of transformations to a program
- be sure that transformations are correct

All transformations are already proved to be correct
A toolbox

Semantic-preserving functional program transformations
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- continuation-passing style transform
  (Plotkin, Steele, Friedman, Wand, Danvy, Filinski)
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- defunctionalization (Reynolds)
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Semantic-preserving functional program transformations

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  (Plotkin, Steele, Friedman, Wand, Danvy, Filinski)
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- explicit control stack introduction (Landin, Danvy)

All transformations are proved to be correct

Each one yields a new adequate representation of the algorithm
This work: inter-derivation between a *recursive descent* and an *abstract machine*.

Our goal is SEC-machine$^4$.

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$^4$Result $\textbf{Stack} \times \textbf{Environment} \times \textbf{Control Stack}$
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- Different formalisms and corresponding implementations might be used, but *equivalence* between them *should be proved*;
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- Functional correspondence by program transformations enables us to derive a family of algorithms for type checking, rather than invent them from scratch;
Summary and conclusion

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- Different formalisms and corresponding implementations might be used, but *equivalence* between them *should be proved*;
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- A tool-chain of transformations is applied to derive those algorithms;
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Summary and conclusion

- Type checking is a computation over a program’s syntax; its semantics may be described in different ways;
- Different formalisms and corresponding implementations might be used, but **equivalence** between them *should be proved*;
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Thank you