From Type Checking by Recursive Descent to Type Checking by Abstract Machine

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LDTA '11
26 March 2011
Saarbrücken, Germany

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This work was carried out while the first author was visiting the BRICS PhD School of Aarhus University in September 2010

What is a program semantics?

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Example of a meaning:

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Example of a meaning: ***types***

Outline

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A parade of semantics

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- *Denotational semantics*: what does a program mean as a mathematical object
  - C. Strachey, D. Scott
- *Operational semantics*: how to compute a program on some abstract machine, what is its result
  - G. Plotkin
- *Axiomatic semantics*: what are properties of the effect of executing a program
  - R.W.Floyd, C.A.R.Hoare

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Operational semantics defines how to *execute a program*.

A diversity of operational semantics

- **Big-step (*natural*) semantics**:
  - program evaluation defined inductively on its syntax
  - computes a *fold* over a program's AST
- ***Big-step abstract machine***
  - Execution traces instead of trees
- ***Small-step operational semantics***
  - Each step: decompose-contract-recompose
- ***Reduction semantics***
  - Contexts and contractions
- ***Small-step abstract machine***
  - Examples: CC, SCC, CK, CEK-machines, Krivine's machine, Landin's SECD etc.

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Related work: Ager-al:PPDP03, Cardelli:TR107, Cousineau-al:SCP87, Danvy:IFL04, Danvy:ICFP08, Felleisen-Friedman:FDPC3, Hannan-Miller:MSCS92, Krivine:04, Landin:CJ64, Launchbury:POPL93, Milne-Strachey:76, Plotkin:JLAP04, Reynolds:ACM72, Sestoft:JFP97, VanHorn-Might:ICFP10...

A diversity of semantic artifacts

Semantics are described via some *meta-languages*

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therefore

Semantic formalisms can be directly represented as ***programs***
(or *semantic artifacts*)

Semantics equivalence problem

All these semantics were developed independently of each other

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Their equivalence should be proved

Semantics equivalence problem

All these semantics were developed independently of each other

Their equivalence should be proved

Can we connect them some other way?

Calculational inter-derivation

Program *semantics* can be connected
via the inter-derivation of the corresponding *semantic artifacts*.[1]

[1] O. Danvy, ICFP '08

This connection has never been done
for *type checking*

Outline

Type checking: description I

Type checking is a semantics of a "typing language" on top of the host language's syntax.

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Type checking is a semantics of a "typing language" on top of the host language's syntax.

Its natural semantics is a familiar one, in the form of *derivation rules*.

$$\text{[t-lam]} \quad \frac{\Gamma[x:\tau_1] \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1 . e : \tau_1 \to \tau_2} \qquad \text{[t-var]} \quad \frac{(x : \tau \in \Gamma)}{\Gamma \vdash x : \tau}$$

$$\text{[t-app]} \quad \frac{\begin{array}{c}\Gamma \vdash e_1 : \tau_1 \to \tau_2 \\ \Gamma \vdash e_2 : \tau_1\end{array}}{\Gamma \vdash e_1 e_2 : \tau_2} \qquad \text{[t-num]} \quad \Gamma \vdash \mathit{number} : \text{num}$$

Type system for the simply typed lambda calculus with numbers

Type checking: description II

Another semantics of typing language: a *small-step abstract machine with control and result stacks* (SEC-machine)[2]

$$
\begin{array}{rcl}
\langle S,\ E,\ num{:}C\rangle & \Rightarrow_t & \langle \mathsf{num}{:}S,\ E,\ C\rangle \\
\langle S,\ E[x{\Rightarrow}\tau],\ x{:}C\rangle & \Rightarrow_t & \langle \tau{:}S,\ E[x{\Rightarrow}\tau],\ C\rangle \\
\langle S,\ E,\ (\lambda x{:}\tau.e){:}C\rangle & \Rightarrow_t & \langle \mathsf{nil},\ E\sqcup\{x{\Rightarrow}\tau\},\ e{:}Lam(\tau,\ S){:}C\rangle \\
\langle S,\ E,\ (e_1 e_2){:}C\rangle & \Rightarrow_t & \langle S,\ E,\ e1{:}Fun(e_2){:}C\rangle \\
\langle \tau_2{:}S,\ E,\ Lam(\tau_1,\ S'){:}C\rangle & \Rightarrow_t & \langle (\tau_1{\rightarrow}\tau_2){:}S',\ E,\ C\rangle \\
\langle (\tau_1{\rightarrow}\tau_2){:}S,\ E,\ Fun(e_2){:}C\rangle & \Rightarrow_t & \langle (\tau_1{\rightarrow}\tau_2){:}S,\ E,\ e_2{:}Arg(\tau_1,\ \tau_2){:}C\rangle \\
\langle \tau_1{:}x{:}S,\ E,\ Arg(\tau_1,\ \tau_2){:}C\rangle & \Rightarrow_t & \langle \tau_2{:}S,\ E,\ C\rangle
\end{array}
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Type checking: description III

And yet another one: *reduction semantics*[3]

$$
\begin{array}{rcl}
e & ::= & n \mid x \mid \lambda x{:}\tau.e \mid e\ e \mid \tau \rightarrow e \mid \textsf{num} \\
T & ::= & T\ e \mid \tau\ T \mid \tau \rightarrow T \mid [\ ] \\
\tau & ::= & \textsf{num} \mid \tau \rightarrow \tau \\
n & ::= & \textit{number}
\end{array}
$$

Hybrid language and type-checking contexts

$$
\begin{array}{rcll}
T[n] & \mapsto_t & T[\textsf{num}] & [\textsf{tc-const}] \\
T[\lambda x{:}\tau.e] & \mapsto_t & T[\tau \rightarrow \{\tau/x\}\ e] & [\textsf{tc-lam}] \\
T[(\tau_1 \rightarrow \tau_2)\ \tau_1] & \mapsto_t & T[\tau_2] & [\textsf{tc-}\tau\beta]
\end{array}
$$

Type-checking reduction rules

[3] G. Kuan, D. MacQueen and R. B. Findler, ESOP '07

Why should we care about different semantics?

Given formalism for a type systems defines a corresponding *semantic artifact*

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Given formalism for a type systems defines a corresponding *semantic artifact*

- Type derivation rules $\sim$ recursive descent
- Machine-like semantics $\sim$ driver-loop abstract machine (CEK, SECD etc.)
- Reduction semantics $\sim$ *decompose-contract-recompose* loop

Why should we care about different semantics?

Given formalism for a type systems defines a corresponding *semantic artifact*

- Type derivation rules $\sim$ recursive descent
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Benefits of non-standard semantics:

- type debugging
- optimized computation

Outline

Semantics equivalence again

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Theorem *[Hankin and Le Métayer]*
(Soundness and Completeness for $\Rightarrow_t$)
$\Gamma \vdash e : \tau$ iff $\langle S, \Gamma, e : C \rangle \Rightarrow_t \langle \tau : S, \Gamma, C \rangle$.

Theorem *[Kuan et al.] (Soundness and Completeness for $\mapsto_t$)*
For any e and τ, $\emptyset \vdash e : \tau$ iff $e \mapsto_t^* \tau$

Our concern

Can we *inter-derive* these semantics **a priori**
rather than *prove* their equivalence **a posteriori?**

Our contribution

Yes, we can.

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Yes, we can.

via *functional inter-derivation*

Transformations instead of proofs

Outline

Two approaches

A mathematician's approach: to prove the equivalence between semantics by induction (or bisimulation by coinduction)

A programmer's approach:

- take one particular implementation
- apply a series of transformations to a program
- be sure that transformations are correct

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A programmer's approach:

- take one particular implementation
- apply a series of transformations to a program
- be sure that transformations are correct

All transformations are already proved to be correct

A toolbox

Semantic-preserving functional program transformations

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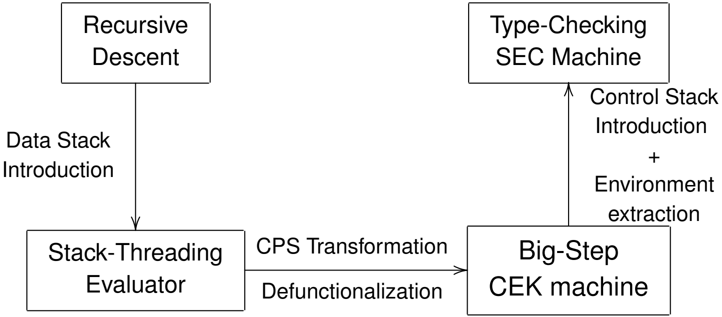
All transformations are proved to be correct

Each one yields a new adequate representation of the algorithm

Inter-derivation

This work: inter-derivation between a *recursive descent* and an *abstract machine*.

Our goal is SEC-machine[4].



[4] *Result* ***S****tack* $\times$ ***E****nvironment* $\times$ ***C****ontrol Stack*

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Thank you